

Name: AKUMA SUNNY . A.

MATRIC NO: 17/ENG04/009

DEPT: ELECTRICAL / ELECTRONICS ENGINEERING

ENG381 ASSIGNMENT II

$$1) y = e^{x^2+x}$$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$$

$$\therefore y'(2x+1) + 2y \\ = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$$

$$\text{But } y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$$

$$\therefore y'' = y'(2x+1) + 2y$$

from the above eqn:

PART A

$$A = y'', A' = y''', A'' = y^{(4)}$$

PART B

$$B = y'(2x+1)$$

$$u = y', u^A = y^{n+1}$$

$$v = 2x+1$$

$$v' = 2$$

$$v''' = 0$$

$$\therefore B^n = (y^{n+1})(2x+1) + 2n(y^n)(2) + 0$$

$$B^n = (2x+1)2y^{n+1} + 2ny^n$$

PART C

$$C = 2y$$

$$C^n = 2y^n$$

$$\therefore A^n = B^n + C^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1) \Rightarrow$$

$$\therefore y^{n+2} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

## Question 2

20)  $y = x^3 e^{4x}$

Using Leibnitz theorem

$$u = e^{4x}, u' = 4e^{4x}, u'' = 16e^{4x}, u''' = 64e^{4x}, u^{(4)} = 256e^{4x}$$

$$v = x^3, v' = 3x^2, v'' = 6x, v''' = 6, v^{(4)} = 0$$

Hence  $u^{(n)} = 4^n e^{4x}$

$$y^{(n)} = u^{(n)}v + \frac{n u^{(n-1)} v'}{2!} + \frac{n(n-1) u^{(n-2)} v''}{3!} + \frac{n(n-1)(n-2) u^{(n-3)} v'''}{4!} + \dots$$

$$y^{(n)} = 4^n e^{4x} (x^3) + \frac{n 4^{n-1} e^{4x} (3x^2)}{2!} + \frac{n(n-1) 4^{n-2} e^{4x} (6x)}{3!} + \frac{n(n-1)(n-2) 4^{n-3} e^{4x} (6)}{4!} + \dots$$

$$y^{(n)} = 4^{n-4} (x)^2 + 4^n x^3 e^{4x} + n 3 \cdot 2x^2 + 4^{n-1} e^{4x} + 3x n(n-1) 4^{n-2} + n(n-1)(n-2) 4^{n-3} e^{4x} + \dots$$

$$y^{(n)} = e^{4x} 4^{n-3} (x^3 + 3nx^2 + 3xn(n-1)4^{4-2} + n(n-1)(n-2) \dots)$$

$$y^{(n)} = e^{4x} 4^{n-3} (64x^3 + 48x^2 + 12xn(n-1) + n(n-1)(n-2) \dots) - (4^n \dots)$$

$$y^{(5)} = e^{4x} 4^{5-3} (64x^3 + 48x^2 + 5) + (2x \cdot 5(5-1) + 5(5-1)(5-2))$$

$$y^{(5)} = e^{4x} 4^2 (64x^3 + 240x^2 + 240x + 8)$$

$$y^{(5)} = 16e^{4x} (64x^3 + 240x^2 + 240x + 8)$$

26)  $x^2 \frac{d^2 y}{dx^2} + n \frac{dy}{dx} + y = 0$

$$x^2 y'' + n y' + y = 0$$

$$w_1 + w_2 + w_3 = 0$$

$$w_1 = x^2 y''$$

$$u = y'', u' = y''', u'' = y^{(4)}, u''' = y^{(5)} \text{ hence, } u^{(n)} = y^{(n+2)}$$

$$v = x^2, v' = 2x, v'' = 2, v''' = 0$$

$$w_1 = u^{(n)}(v) + \frac{n u^{(n-1)} v'}{2!} + \frac{n(n-1) u^{(n-2)} v''}{3!} + \frac{n(n-1)(n-2) u^{(n-3)} v'''}{4!} + \dots$$

$$w_1 = y^{(n+2)} (2x^2) + n y^{(n+1)} (2x) + \frac{n(n-1)}{2} y^{(n)} (2) + 0 \dots$$

$$w_1 = x^2 y^{(n+2)} + 2x n y^{(n+1)} + y^{(n)} n(n-1)$$

$$w_2 = xy'$$

$$u = y', u' = y'', u'' = y''' \text{ hence } u'' = y^{n+1}$$

$$v = 1 \quad v' = 0 \quad v'' = 0$$

$$w_2 = y^{n+1}(x) + n y^n(1) + 0$$

$$w_2 = x y^{n+1} + n \cdot y^n$$

$$w_3 = y$$

$$u = y \quad u' = y' \text{ hence } u^n = y^n$$

$$v = 1 \quad v' = 0$$

$$w_3 = u^n(v) + n u^{n-1}(v')$$

$$= y^n(1) + 0 = y^n$$

$$w_1 + w_2 + w_3 = 0$$

$$y^{n+2}(n^2) + n y^{n+1}(2x) + n(n-1)y^n + x y^{n+1} + n y^n + y^n = 0$$

$$x^2 y^{n+2} + x y^{n+1}(2x+1) + y^n(n(n-1) + n + 1)$$

$$x^2 y^{n+2} + x y^{n+1}(2n+1) + y^n(n^2+1) = 0$$