$(5\cdot 2)\, 4^{(5-3)} e^{4x}$

$y^5 = 4^5 x^3 e^{4x} + 15x^2 4^4 e^{4x} + 60x4^3 e^{4x} + 60 \cdot 4^2 e^{4x}$

$= 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}$

$\therefore y^5$ of $x^3 e^{4x} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$w_1 = x^2 y''$

$u = y''$	$v = x^2$
$u^n = y^{(n+2)}$	$v' = 2x$
$u^{(n-1)} = y^{(n+1)}$	$v'' = 2$
$u^{(n-2)} = y^n$	$v''' = 0$

$w_2 = xy'$

$u = y'$	$v = x$
$u^n = y^{(n+1)}$	$v' = 1$
$u^{(n-1)} = y^n$	$v'' = 0$

$w_3 = y$

$y^{(n)}$

for w_1

$= y^{(n+2)} \cdot x^2 + n \cdot y^{(n+1)} \cdot 2x + \frac{n(n-1)}{2!} \cdot y^n \cdot 2$

$= x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n$

for w_2

$= y^{(n+1)} \cdot x + n \cdot y^{(n)} \cdot 1$

$= xy^{(n+1)} + ny^{(n)}$

for w_3

$= y^n$

$\therefore x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$

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17/ENG04/068
ENG 381 Assignment II

1. If $y = e^{x^2+x}$
show that
$y'' = y'(2x+1) + 2y$
and, hence, prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2. Using the Leibnitz theorem, given that
(i) $y = x^3 e^{2x}$, determine $y^{(5)}$
(ii) $x^2 \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$, show that
$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Solution

$y = e^{x^2+x}$
$y' = (2x+1)e^{x^2+x}$
$y'' = (2x+1)(2x+1)e^{x^2+x} + 2 \cdot e^{x^2+x}$
$y'' = (2x+1)y' + ~~2e^{x^2+x}~~ 2y$
$y'' = y'(2x+1) + 2y$

Letting $\omega_1 = y'(2x+1)$
$\omega_2 = 2y$

ω_1
$u = y'$ $\quad$ $v = 2x+1$
$u' = y^2$ $\quad$ $v' = 2$
$u^n = y^{(n+1)}$ $\quad$ $v'' = 0$

ω_2
$u = y$ $\quad$ $v = 2$
$u^n = y^n$ $\quad$ $v' = 0$

for w_1

$= y^{(n+1)}(2x+1) + ny^{(n+1-1)} \cdot 2 + \frac{n(n-1)}{1\times 2} y^{(n+1-2)} \cdot 0$

$= y^{(n+1)}(2x+1) + ny^{(n)}(2)$

for w_2

$y^{n} 2 + ny^{(n-1)} \cdot 0$

$= y^{(n)} \cdot 2$

Adding w_1 and w_2

$= y^{(n+1)}(2x+1) + ny^{(n)} 2 + y^{n} \cdot 2$

$= y^{(n+1)}(2x+1) + 2ny^{(n)} + 2y^{(n)}$

$= y^{(n+1)}(2x+1) + 2y^{n}(n+1)$

then,

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{n}(n+1)$

$= y^{(n+1)}(2x+1) + 2(n+1)y^{n}$

therefore

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2(n+1)y^{n}$

2. (b) $y = x^3 e^{4x}$

$u = e^{4x}$	$v = x^3$
$u' = 4e^{4x}$	$v' = 3x^2$
$u^2 = 16e^{4x}$	$v'' = 6x$
$u^n = 4^n e^{4x}$	$v''' = 6$
$u^{(n-1)} = 4^{(n-1)} e^{4x}$	$v^{iv} = 0$
$u^{(n-2)} = 4^{(n-2)} e^{4x}$	
$u^{(n-3)} = 4^{(n-3)} e^{4x}$	

$= 4^n e^{4x} \cdot x^3 + n \cdot 4^{(n-1)} e^{4x} \cdot 3x^2 + n(n-1) \cdot 4^{(n-2)} e^{4x} \cdot 6x^3 + \dots$

$\dots \frac{n(n-1)(n-2)}{1\times 2\times 3} \cdot 4^{(n-3)} e^{4x} \cdot 6 + \dots$

$= 4^n x^3 e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + 3n(n-1)x 4^{(n-2)} e^{4x} + \dots$

$\dots n(n-1)(n-2) 4^{(n-3)} e^{4x}$

Recall $y^{n} = y^{5}$

$y^5 = 4^5 x^3 e^{4x} + 3(5)x^2 4^{(5-1)} e^{4x} + 3(5)(5-1)x 4^{(5-2)} e^{4x} + 5(5-1)\dots$

$$= x^2y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^{(n)} + ny^{(n)} + y^{(n)} = 0$$

$$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + [n(n-1) + n + 1]y^{(n)} = 0$$

$$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + (n^2 - n + n + 1)y^{(n)} = 0$$

$$= x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$