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ENG 381

1.  $y = e^{x^2+x}$   
 $y' = (2x+1)e^{x^2+x}$   
 $y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$   
 $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$   
 $\therefore y'(2x+1) + 2y$   
 $= (2x+1)e^{x^2+x} + 2e^{x^2+x}$   
But  $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$   
 $\therefore y'' = y'(2x+1) + 2y$   
From the above equation;

Part A

$$A = y'', \quad A' = y''', \quad A'' = y^{(5+A)}$$

Part B

$$B = y'(2x+1)$$

$$u = y', \quad u^A = y^{n+1}$$

$$v = 2x+1$$

$$v' = 2$$

$$v''' = 0$$

$$\therefore B^n = (y^{n+1})(2x+1) + 2n(y^n)(2) + 0$$

$$B^n = (2x+1)2y^{n+1} + 2ny^n$$

Part C

$$C = 2y$$

$$C^n = 2y^n$$

$$A^n = B^n + C^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2y^n \Rightarrow \text{proven}$$

$$\therefore y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

## Question 2

2a)  $y = x^3 e^{4x}$

Using Leibnitz theorem  
 $u = e^{4x}, u' = 4e^{4x}, u'' = 16e^{4x}, u''' = 64e^{4x}, u^{IV} = 256e^{4x}$   
 $v = x^3, v' = 3x^2, v'' = 6x, v''' = 6, v^{IV} = 0$

Hence  $u^n = f^n e^{4x}$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v'''$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^{IV} + \dots$$

$$y^n = 4^n e^{4x} (x^3) + n 4^{n-1} e^{4x} (3x^2) + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} (6x) + \dots$$

$$\frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} (6) + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4} (0)^2 + \dots$$

$$4^{n-3} e^{4x} + 0$$

$$y^n = 4^n x^3 e^{4x} + n 3x^2 4^{n-1} e^{4x} + 3x n(n-1) 4^{n-2} e^{4x} + n(n-1)(n-2) 4^{n-3} e^{4x} + 0$$

$$y^n = e^{4x} 4^{n-3} (3x^3 4^3 + 3n x^2 4^2 + 3x n(n-1) 4^1 + n(n-1)(n-2))$$

$$y^n = e^{4x} 4^{n-3} (64x^3 + 48x^2 n + 12x n(n-1) + n(n-1)(n-2)) = (y^n)$$

$$y^5 = e^{4x} 4^{5-3} (64x^3 + 48x^2(5) + (12x \cdot 5(5-1) + 5(5-1)(5-2)))$$

$$y^5 = e^{4x} 4^2 (64x^3 + 240x^2 + 240x + 36)$$

$$y^5 = 16e^{4x} (64x^3 + 240x^2 + 240x + 36)$$

2b)  $x^2 \frac{d^2 y}{dx^2} + n \frac{dy}{dx} + y = 0$

$$x^2 y'' + n y' + y = 0$$

$$w_1 + w_2 + w_3 = 0$$

$$w_1 = x^2 y''$$

$$u = y'', u' = y''', u'' = y^{IV}, u''' = y^V \text{ hence, } u^n = y^{n+2}$$

$$v = x^2, v' = 2x, v'' = 2, v''' = 0$$

$$w_1 = u^n (v) + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} (v'') + \frac{n(n-1)(n-2)}{3!} u^{n-3} (v''') + \dots$$

$$u^{n-3} v^{iii} + \dots$$

$$w_1 = y^{n+2} (2x^2) + ny^{n+1} (2x) + \frac{n(n-1)}{2} y^n (2) + 0$$

$$w_1 = x^2 y^{n+2} + 2xny^{n+1} + y^n n(n-1)$$

$$w_2 = xy'$$

$$u = y', \quad u' = y'', \quad u'' = y''', \quad \text{Hence } u^n = y^{n+1}$$

$$v = x, \quad v' = 1, \quad v'' = 0$$

$$w_2 = y^{n+1} (x) + ny^n (1) + 0$$

$$w_2 = xy^{n+1} + ny^n$$

$$w_3 = y$$

$$u = y, \quad u' = y', \quad \text{Hence } u^n = y^n$$

$$v = x, \quad v' = 1, \quad v'' = 0$$

$$w_3 = y^{n+1} (x) + ny^n (1) + 0$$

$$w_3 = xy^{n+1} + ny^n$$

$$w_3 = y$$

$$u = y, \quad u' = y', \quad \text{Hence } u^n = y^n$$

$$v = x, \quad v' = 1$$

$$w_3 = u^n (v) + nu^{n-1} (v')$$

$$= y(1) + 0 = y$$

$$w_1 + w_2 + w_3 = 0$$

$$y^{n+2} (n^2) + ny^{n+1} (2x) + n(n-1) y^n + xy^{n+1} + ny^n + y^n = 0$$

$$x^2 y^{n+2} + 2xy^{n+1} (2n+1) + y^n (n(n-1) + n+1)$$

$$x^2 y^{n+2} + 2xy^{n+1} (2n+1) + y^n (n^2+1) = 0$$