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17/SCI14/013

CIVIL ENGINEERING

1) If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

Soln

$y^0 = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$ $\quad y'' = (2x+1)(2x+1)e^{x^2+x} + \cancel{2e^{x^2+x}}\ 2e^{x^2+x}$

$\therefore y'' = (2x+1)y' + 2(y)$

Note: Recall $y' = (2x+1)e^{x^2+x}$ \$ $y = e^{x^2+x}$

(ii) $y^0 = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)y' + 2y$

$y^n = (2x+1)y^{n-1} + 2(n-1)y^{n-2}$

$y^{n+2} = (2x+1)y^{n+2-1} + 2(n+1)y^{n+2-2}$

$\therefore y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

(2) $y = x^3 e^{4x}$

$v = x^3 \quad v' = 3x^2 \quad v'' = 6x \quad v''' = 6 \quad v^{iv} = 0$

$u = e^{4x} \quad u' = 4e^{4x} \quad u'' = 16e^{4x} \quad u''' = 64e^{4x} \quad u^{iv} = 256e^{4x}$

$y^n = 4^n e^{4x}$

$$y = u^n v^0 \; n u^{(n-1)} v' + \frac{n(n-1)}{2} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{2\times3} u^{n-3} v^3 + \frac{n(n-1)(n-2)(n-3)}{2\times3\times4} u^{(n-4)} v^4$$

$y^5 \; \cancel{4^5 x^3}$

$$y^5 = 4^5 e^{4x} x^3 + 5\cdot 256 e^{4x} \cdot 3x^2 + \frac{5(4)}{2} 64e^{4x} \cdot 6x + \frac{5(4\times3)}{2\times3} 16e^{4x} \cdot 6 + 0$$

$$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}$$

$$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$W_1 = x^2 y''$

$W_2 = xy'$

$W_3 = y$

Using W_1

$V = x^2 \quad v' = 2x \quad v'' = 2 \quad v''' = 0$

$u = y'' \quad u' = y''' \quad u'' = y^{iv} \quad u^3 = y^v$

$u^n = y^{n+2}$

$W_2 \Rightarrow v = x \quad v' = 1 \quad v^2 = 0$

$u = y' \quad u' = y'' \quad u'' = y'''$

$u^n = y^{n+1}$

$W_3 = y$

$V = 1 \quad v' = 0$

$u = y \quad u' = y'$

$u^n = y^n$

$$H_1^n = u^n v^0 + n u^{n-1} v^1 + \frac{n(n-1)}{2} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{2\times 3} u^{n-3} v$$

$$= y^{n+2}(x^2) + n y^{n+1}(2x) + \frac{n(n-1)}{\cancel{2}} y^n (\cancel{2}) + 0$$

$$= y^{n+2} x^2 + 2xn y^{n+1} + n(n-1) y^n$$

$$H_2^n = y^{n+1}(x) + n y^n (1) + 0$$

$$= x y^{n+1} + n y^n$$

$$H_3^n = y^n (1) + 0$$

$$= y^n$$

$$H_1 + H_2 + H_3$$

$$= y^{n+2} x^2 + 2xn y^{n+1} + n(n-1) y^n + x y^{n+1} + n y^n + y^n$$

$$= x^2 y^{n+2} + (2n+1) x y^{n+1} + (n^2 - n + n - 1) y^n$$

$$= x^2 y^{n+2} + (2n+1) x y^{n+1} + (n^2 - 1) y^n$$