OGBONNA WISDOM OKORO OGONNA
17/ENG08/037
CIVIL ENGINEERING DEPT

1 If $y = e^{x^2+x}$,

show that

$$y'' = y'(2x+1) + 2y$$

and hence prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

$y^0 = e^{x^2+x}$ $y' = (2x+1)e^{x^2+x}$ $y'' = (2x+1)(2x+1)e^{x+x} + 2e^{x^2+x}$

$y^0 = e^{x^2+x}$

$y' = (2x+1)y^0$

$y'' = (2x+1)y' + 2y$

$y'' = (2x+1)y' + 2(y)$

QED

$y^n = (2x+1)y^{n-1} + 2(n-1)y^{n-2}$

$y^{n+2} = (2x+1)y^{n+2-1} + 2y^{(n+1)_{n+2-2}}$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

Q.E.D

2 i) $y = x^3e^{4x}$, determine $y^{(5)}$

$v^0 = x^3$ $v' = 3x^2$ $v'' = 6x$ $v''' = 6$ $v^{iv} = 0$

$u^0 = e^{4x}$ $u' = 4e^{4x}$ $u'' = 16e^{4x}$ $u''' = 64e^{4x}$ $u^{iv} = 256e^{4x}$

$u^n = 4^n e^{4x}$

$$y^n = u^0v^{(n)} + nu^{(n-1)}v^{(1)} + \frac{n(n-1)}{2}u^{(n-2)}v^{(2)} + \frac{n(n-1)(n-2)}{2\cdot 3}u^{(n-3)}v^3 + \frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}u^{(n-4)}\cdot v^+$$

$$y^5 = 1024e^{4x}\cdot x^3 + 5\cdot 256e^{4x}\cdot 3x^2 + \frac{5(4)}{2}\cdot 64e^{4x}\cdot 6x + \frac{5(12)}{6}\cdot 16e^{4x}\cdot 6 + \frac{5\cdot 12\cdot 2}{6\cdot 4}\cdot 4e^{4x}\cdot 0$$

$$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$$

$$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

11) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$w_1 = x^2\frac{d^2y}{dx^2}$

$v^0 = x^2 \quad v' = 2x \quad v'' = 2 \quad v''' = 0$

$u = y'' \quad u' = y''' \quad u'' = y^{iv} \quad u''' = y^{v}$

$u^n = y^{n+2}$

$\therefore w_1^n = y^{n+2}x^2 + n y^{n+1}\cdot 2x + \frac{n(n-1)}{2} y^n \cdot 2 + \frac{n(n-1)(n-2)}{2\cdot 3} y^{n-1}\cdot 0$

$w_1^n = y^{n+2}x^2 + 2xn y^{n+1} + n(n-1)y^n$

$w_2 = x\frac{dy}{dx}$

$v = x \quad v' = 1 \quad v'' = 0$

$u = \frac{dy}{dx} \quad u' = y'' \quad u'' = y'''$

$u^n = y^{n+1}$

$\therefore w_2^n = y^{n+1}x + n\cdot y^n \cdot 1 + 0$

$w_2^n = xy^{n+1} + ny^n$

$w_3 = y$

$v^0 = 1 \quad v' = 0$

$u = y \quad u' = y'$

$u^n = y^n$

$\therefore w_3^n = y^n \cdot 1 + 0$

$w_3^n = y^n$

$w_1 + w_2 + w_3 = 0$

$y^{n+2}x^2 + 2xny^{n+1} + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$

$x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2 - n + n + 1)y^n = 0$

$x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$

QED