Bakare Sharafadeen omogbolahan.
17/Engr/024
ENG 381
Assignment 2.

① If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$ And hence Prove that: $y^{(n+2)} = 2x+1y^{(n+1)} + 2(n+1)y^n$.

Solution.

$y = e^{x^2+x}$ — i $\frac{dy}{dx} = y' = (2x+1) \cdot e^{x^2+x}$ — — — ii

$\frac{d^2y}{dx^2} = y'' = [(2x+1) \cdot (2x+1) e^{x^2+x}] + e^{x^2+x} \cdot 2$ — — (iii)

Sub eqn i and ii into eqn iii we have:

$y'' = (2x+1)y' + 2y$

If $w_1 = y''$

$v = 1 \quad v' = 0$

$u = y'', u' = y''' \therefore u^n = y^{n+2}$

$w_1^n = u^n v + n u^{n-1} v'$

$w_2^n = y^{n+2} + n u^{n+1} \cdot 0 = y^{n+2}$.

If $w_2 = (2x+1)y'$, $v = 2x+1$, $v^2 = 2$, $v^3 = 0$

$u = y' \therefore u^n = y^{n+1}$

$w_2^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2$.

$= y^{n+1} \cdot (2x+1) + n y^n \cdot 2 + \frac{n(n-1)}{2} y^{n-1} \cdot 0$

$w_2^n = (2x+1) y^{n+1} + 2n y^n$.

If $w_3 = 2y$ $\quad v = 2 \quad v' = 0$

$u = y \quad u^n = y^n$.

$w_3^n = u^n v + n u^{n-1} v'$

$= y^n \cdot 2 + \cancel{n y^{n-1} \cdot 0} = 2y^n$.

$w_1^n = w_2^n + w_3^n$.

$y^{n+2} = (2x+1)\, y^{n+1} + 2n y^n + 2y^n$

$y^{(n+2)} = (2x+1)\, y^{(n+1)} + 2(n+1)\, y^n$ //.

(2) Using Leibnitz theorem, given that.

(i) $y = x^3 e^{4x}$ determine $y^5$

Soln

$y = x^3 e^{4x}$ $\therefore v = x^3,\ v' = 3x^2,\ v^2 = 6x,\ v^3 = 6,\ v^4 = 0.$

$u = e^{4x},\ u' = 4e^{4x};\ u^2 = 16e^{4x},\ u^3 = 64e^{4x}.$

$\therefore u^n = 4^n e^{4x}.$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3\cdot 2} u^{n-3} v^3 +$$

$$\frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^4$$

$$y^n = 4^n e^{4x}\cdot x^3 + n 4^{n-1} e^{4x}\cdot 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x}\cdot 6x +$$

$$\frac{n(n-1)(n-2)}{3\cdot 2} 4^{n-3} e^{4x}\cdot 6 + \cancel{\frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4} e^{4x}} - 0$$

$y^n = x^3 4^n e^{4x} + n 3x^2 4^{n-1} e^{4x} + \cdot 3x + n(n-1)(n-2)\, 4^{n-3} e^{4x}.$

$y^n = 4^{n-3} e^{4x}\left(4^3 x^3 + n4\, 3x^2 + n(n-1)4\cdot 3x + n(n-1)(n-2)\right.$

$y^5 = 4^{5-3} e^{4x}\left[4^3 x^3 + 5\cdot 4^2\cdot 3x^2 + 5(5-1)\, 4\cdot 3x + 5(5-1)(5-2)\right]$

$y^5 = 16 e^{4x}\left[64x^3 + 240x^2 + 240x + 60\right]$ //.

ii $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that

$x^2 y^{(n+2)} + (2n+1)x\, y^{(n+1)} + (n^2+1)\, y^n = 0.$

Soln

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0 \Rightarrow x^2 y'' + x y' + y = 0.$

If $w_1 = x^2 y''$ $\therefore v = x^2,\ v^2 = 2x;\ v^3 = 2,\ v^3 = 0$

$u = y'',\ u' = y''',\ \therefore u^n = y^{n+2}.$

$$w_1^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2 + \frac{n(n-1)(n-2) u^{n-3} v^3}{3 \times 2.}$$

$$w_1 = y^{n+2} \cdot x^2 + n\, y^{n+1} \cdot 2x + \frac{(n)(n-1)}{2} y^n \cdot 2 + \cancel{\frac{n(n-1)(n-2)}{3 \times 2}} y^{n-1} \cdot 0$$

$$w_1 = x^2 y^{n+2} + 2xn\, y^{n+1} + (n)(n-1) y^n.$$

$w_2 = x y'$ $\therefore$ $v = x$ $v' = 1$ $v^2 = 0$

$u = y'$, $u' = y''$, $u^n = y^{n+1}$

$$w_2^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2$$

$$w_2^n = y^{n+1} \cdot x + n y^n \cdot 1 \quad \frac{n(n-1)^2}{2} y^{n-1} \cdot 0$$

$$w_2 = x y^{n+1} + n y^n$$

If $w_3 = y$ $\therefore$ $v = 1$, $v' = 0$

$u = y$, $u^n = y^n.$

$$w_3^n = u^n v + n u^{n-1} v'$$

$$= y^n \cdot 1 + n y^{n-1} \cdot 0 = y^n$$

$\therefore$ $w_1^n + w_2^n + w_3^n = 0.$

$$\Rightarrow x^2 y^{n+2} + 2xn\, y^{n+1} + n(n-1) y^n + x y^{n+1} + n y^n + y^n.$$

$$\Rightarrow x^2 y^{(n+2)} + (2n+1) x\, y^{(n+1)} + n^2 + 1\, y^n = 0 \quad //.$$