

$$y = e^{x^2+x}$$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$$

$$\therefore y'(2x+1) + 2y$$

$$= (2x+1)e^{x^2+x} \cdot (2x+1) + 2(e^{x^2+x})$$

$$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$$\therefore y'' = y'(2x+1) + 2y,$$

from the above equation,

Part A,

$$A = y'', \quad A' = y''', \quad A^n = y^{2+n}$$

Part B

$$B = y'(2x+1)$$

$$u = y'$$

$$u^n = y^{n+1}$$

$$v = 2x+1$$

$$v' = 2$$

$$v''' = 0$$

$$\therefore B^n = (y^{n+1})(2x+1) + 2n(y^n)(2) + 0$$

$$B^n = (2x+1)y^{n+1} + 2ny^n$$

Part C,

$$C = 2y$$

$$C^n = 2y^n$$

$$\therefore A^n = B^n + C^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2y^n (n+1)$$

$$\therefore y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n //$$

2. i) $y = x^3 e^{4x}$, $y^{(5)}$

let $u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u^{(n)} = 4^n e^{4x}$
 let $v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v^{(4)} = 0$

By Leibniz theorem,

$$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} \cdot 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} \cdot 4^{n-3} e^{4x} \cdot 6 + 0$$

$$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 n \cdot 4^{n-1} e^{4x} + 3n(n-1) \cdot 4^{n-2} e^{4x} x + n(n-1)(n-2) \cdot 4^{n-3} e^{4x}$$

$$\therefore y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2(5) \cdot 4^4 e^{4x} + 3(5)(4) \cdot 4^3 e^{4x} x + (5)(4)(3) \cdot 4^2 e^{4x}$$

$$y^5 = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} \cdot x + 960 e^{4x}$$

$$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15) //$$

v) $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^n = 0$

for Part A,
 $A = x^2 y''$

$u = y''$, $u^n = y^{n+2}$
 $v = x^2$, $v' = 2x$, $v'' = 2$, $v^{(3)} = 0$

$$A^n = y^{(n+2)} x^2 + n(y^{(n+1)}) \cdot 2x + \frac{n(n-1)}{2!} \cdot (y^n) \cdot 2 + 0$$

$$A^n = x^2 y^{(n+2)} + 2x n y^{(n+1)} + n(n-1) y^n$$

for Part B,
 $B = xcy'$

$$u = y', \quad u^n = y^{n+1}$$
$$v = x, \quad v' = 1, \quad v'' = 0$$

$$B^n = (y^{n+1}) \cdot x + n(y^n) \cdot 1 \neq 0$$
$$= xy^{(n+1)} + ny^n$$

for Part C

$$C = y$$
$$C^n = y^n$$

$$\therefore A^n + B^n + C^n = 0$$

$$= x^2 y^{(n+2)} + 2xny^{(n+1)} + (n^2 - n)y^n + xy^{(n+1)} + ny^n + y^n = 0$$

$$= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$$

$$= x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2 + 1)y^n = 0 //$$