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1 $y = e^{x^2+x}$

$y = (2x+1)e^{x^2+x}$

$y'' = 2x+1 \frac{d}{dx}(e^{x^2+x}) + e^{x^2+x}\frac{d}{dx}(2x+1)$ [Product rule]

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$ ∴ Proven

1b $y'' - y'(2x+1) - 2y = 0$

using leibnitz theorem

$w' = y''$

$w_2 = y'(2x+1)$

$2w_3 = 2y$

degenerate equ $= w_1 - w_2 - w_3 = 0$

w_1

$u = y''$ $u' = y'''$ hence $u^n = y^{n+2}$

$v = 1$ $v' = 0$

$w_1 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \dots$

$w_1 = y^{n+2}(1) + n y^{n+1}(0)$

$w_1 = y^{n+2}$ ... (1)

w_2

$w_2 = y'(2x+1)$

$u = y'$ $u' = y''$ $u''' = y''''$ Hence $u^n = y^{n+1}$

$v = 2x+1$ $v' = 2$ $v'' = 0$

$w_2 = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3$

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$$w_2 = y^{n+1}(2x+1) + n y^n (2) + \frac{n(n-1)}{2!} y^{n-2}(0) + \ldots$$
$$w_2 = y^{n+1}(2x+1) + 2n y^n + 0$$
$$w_2 = y^{n+1}(2x+1) + 2n y^n$$

w_3

$u = y \quad u' = y'$ Hence $u^n = y^n$

$v = 2 \quad v' = 0$

$$w_3 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \ldots$$
$$w_3 = y^n (2) + n y^{n-1}(0) + \ldots$$
$$w_3 = 2y^n \ldots$$

Putting back into the degenerate equation

$$w_1 - w_2 - w_3 = 0$$
$$y^{n+2} - (y^{n+1}(2x+1) + 2n y^n) - 2y^n = 0$$
$$y^{n+2} - y^{n+1}(2x+1) + 2n y^n - 2y^n = 0$$

$$y^{n+2} = y^{n+1}(2x+1) + 2n y^n + 2y^n =$$
$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$$
$$y^{n+2} = y^{n+1}$$

Question 2

2.) $y = x^2 e^{4x}$

using leibnitz theorem

$u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$ $u''' = 64e^{4x}$ $u^{IV} = 256e^{4x}$

$v = x^3$ $v' = 3x^2$ $v'' = 6x$ $v''' = 6$ $v^{IV} = 0$

Hence $u^n = 4^n e^{4x}$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3$

$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^2 + \ldots$

$y^n = 4^n e^{4x}(x^3) + n 4^{n-1} e^{4x}(3x^2) + \frac{n(n-1)}{2!} 4^{n-2} e^{4x}(6x)$

$+ \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4} (0)^2$

$y^n = 4^n x^3 e^{4x} + n 3x 4^{n-1} e^{4x} + 3x n(n-1) 4^{n-2} + n(n-1)(n-2) 4^{n-3} e^{4x} + \ldots$

$y^n = e^{4x} 4^{n-3} (x^3 4^3 + 3n x^2 4^2 + 3xn(n-1)4 + n(n-1)(n-2))$

$y^n = e^{4x} 4^{n-3} (64x^3 + 48x^2 n + 12x n(n-1) + n(n-1)(n-2))$

$y^5 = e^{4x} 4^2 (64x^3 + 240x^2 + 240x + 36)$

$y^5 = 16e^{4x} (64x^3 + 240x^2 + 240x + 36)$

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26) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$w_1 + w_2 + w_3 = 0$

$w_1 = x^2 y''$

$u = y''$, $u' = y'''$ $u'' = y^{iv}$ $u''' = y^{v}$ $u^n = y^{n+2}$

$v = x^2$ $v' = 2x$ $v'' = 2$ $v''' = 0$

$w_1 = u^n(v) + nu^{n-1}v' + \frac{n(n-1)}{2!} u^{n-2}v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3}v''' + -$

$w_1 = y^{n+2}(x^2) + ny^{n+1}(2x) + \frac{n(n+1)}{2} y^n(2) + 0$

$w_1 = x^2y^{n+2} + 2xny^{n+1} + \frac{2}{2} y^n (n-1)$

$w_2 = xy'$

$u = y'$ $u' = y''$ $u'' = y'''$ $u^n = y^{n+1}$

$v = x$ $v' = 1$ $v'' = 0$

$w_2 = y^{n+1}(x) + ny^n(1) + 0$

$w_2 = xy^{n+1} + ny^n$

$w_3 = y$

$u = y$ $u' = y'$ Hence $u^n = y^n$

$v = 1$ $v' = 0$

$w_3 = u^n(v) + nu^{n-1}(v')$

$= y(1) + 0 = y$

$w_1 + w_2 + w_3 = 0$

$y^{n+2}(x^2) + ny^{n+1}(2x) + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n(n-1)+n)$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2+1) = 0$