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 COURSE - ENIG 381  
 MAT NO - 171ENG061029.

Answer.

$$y = e^{x^2+x}$$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$$

$$\therefore y'(2x+1) + 2y$$

$$= (2x+1)^2 e^{x^2+x} + 2(e^{x^2+x})$$

$$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$$

$$\text{but } y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$$

$$\therefore y'' = y'(2x+1) + 2y$$

From the above equation.

Part A.

$$A = y'' \quad A' = y''' \quad A^n = y^{2+n}$$

Part B.

$$B = y'(2x+1)$$

$$u = y' \quad u^n = y^{n+1}$$

$$v = 2x+1$$

$$v' = 2$$

$$v^{2+n} = 0$$

$$\therefore B_n = (y^{n+1})(2x+1) + n(y^n)(2)$$

$$B_n = (2x+1)y^{n+1} + 2ny^n$$

Part C.

$$C = 2y$$

$$C^n = 2y^n$$

$$\therefore A^n = B^n + C^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$$

$$\therefore y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$$

2.  $y = x^3 e^{4x}$  determine  $y^{(5)}$

$$\text{Let } u = e^{4x} \quad u' = 4e^{4x} \quad u'' = 16e^{4x} \quad u^n = 4^n e^{4x}$$

$$v = x^3 \quad v' = 3x^2 \quad v'' = 6x \quad v''' = 6 \quad v^{(n)} = 0$$

By Leibniz theorem

$$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} \cdot 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} \cdot 4^{n-3} e^{4x} \cdot 6$$

$$(n-2) \cdot 4^n e^{4x}$$

$$y^n = 4^n e^{4x} x^3 + 3x^3 n \cdot 4^{n-1} e^{4x} + 3n(n-1) \cdot 4^{n-2} e^{4x} x + n(n-1)(n-2) \cdot 4^{n-3} e^{4x}$$

$$y^5 = 4^5 e^{4x} x^3 + 3x^3 (5) \cdot 4^4 e^{4x} + 3(5)(4) \cdot 4^3 e^{4x} x + (5)(4)(3) \cdot 4^2 e^{4x}$$

$$y^5 = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 960 e^{4x}$$

$$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$$

ii.  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$ . Show that

$$x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^n = 0$$

For part A.

$$A = x^2 y''$$

$$u = y'' \quad u^n = y^{n+2}$$

$$v = x^2 \quad v' = 2x \quad v'' = 2 \quad v''' = 0$$

$$A^n = (y^{n+2}) x^2 + n(y^{n+1}) \cdot 2x + n(n-1) \cdot y^n \cdot 2 + 0$$

2!

$$A^n = x^2 y^{(n+2)} + 2x n y^{(n+1)} + n(n-1) y^n$$

For Part B.

$$B = x y'$$

$$u = y' \quad u^n = y^{n+1}$$

$$v = x \quad v' = 1 \quad v'' = 0$$

$$B^n = (y^{n+1}) \cdot x + n(y^n) \cdot 1 + 0$$

$$= x y^{(n+1)} + n y^n$$

For part C.

$$C = y$$

$$C^n = y^n$$

$$A^n + B^n + C^n = 0$$

$$x^2 y^{(n+2)} + 2x n y^{(n+1)} + (n^2 - n) y^n + x y^{(n+1)} + n y^n + y^n = 0$$

$$= x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^n (n^2 - n + n + 1) = 0$$

$$= x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$$