

$$y^n = 4^n e^{4x} x^3 + 3n \cdot 4^{n-1} x^2 e^{4x} + 3n(n-1) 4^{n-2} e^{4x} x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$y^5 = 4^5 e^{4x} x^3 + 3x^2(5) 4^{5-1} e^{4x} + 3(5)(4) 4^{5-2} e^{4x} x + 5(4)(3) 4^{5-3} e^{4x}$$

$$y^5 = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 960 e^{4x}$$

$$I^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$$

2ii  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$  show that  $x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^{(n)} = 0$

sol.

$$A = x^2 y''$$

$$u = y'', u' = y'''$$

$$v = x^2, v' = 2x, v'' = 2, v''' = 0$$

$$A^n = (y''') x^2 + n(y''') 2x + \frac{n(n-1)}{2} (y'') x + 1 \cdot 0$$

$$A^n = x^2 y^{(n+2)} + 2x n y^{(n+1)} + n(n-1) y^{(n)}$$

$$B = x y'$$

$$u = y', u' = y''$$

$$v = x, v' = 1, v'' = 0$$

$$B^n = (y'') x + n(y'') \cdot 1 + 0$$

$$= x y^{(n+1)} + n y^{(n)}$$

$$C = y$$

$$C^n = y^n$$

$$A^n + B^n + C^n = 0$$

$$= x^2 y^{(n+2)} + 2x n y^{(n+1)} + (n^2 - n) y^{(n)} + x y^{(n+1)} + n y^{(n)} + y^n = 0$$

$$= x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^{(n)} (n^2 - n + n + 1) = 0$$

$$= x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$$

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$$y = e^{2x}$$

$$y' = (2x+1)e^{2x}$$

$$y'' = 2e^{2x} + (2x+1)(2x+1)e^{2x}$$

$$y'' = 2e^{2x} + (2x+1)^2 e^{2x}$$

$$y'(2x+1) + 2y$$

$$= (2x+1)e^{2x} \cdot (2x+1) + 2(e^{2x})$$

$$= (2x+1)^2 e^{2x} + 2e^{2x}$$

but  $y'' = 2e^{2x} + (2x+1)^2 e^{2x}$

$$y'' = y'(2x+1) + 2y$$

$$A = y, \quad A' = y', \quad A'' = y''$$

$$B = y(2x+1)$$

$$u = y', \quad v = y''$$

$$v = 2x+1$$

$$v' = 2$$

$$v'' = 0$$

$$B'' = (y''')(2x+1) + 2(y'')(2) + 0$$

$$B'' = (2x+1)y''' + 2y''$$

$$C = 2y$$

$$C' = 2y'$$

$$A'' = B'' + C'$$

$$y'''' = (2x+1)y'''' + 2y'' + 2y'$$

$$y'''' = (2x+1)y'''' + 2y'' + 2y'$$

$$y'''' = (2x+1)y'''' + 2(2x+1)y''$$

2i.  $y = x^3 e^{4x}$ , determine  $y^{(5)}$ .

let  $u = e^{4x}$ ,  $u' = 4e^{4x}$ ,  $u'' = 16e^{4x}$ ,  $u''' = 4^3 e^{4x}$ .

let  $v = x^3$ ,  $v' = 3x^2$ ,  $v'' = 6x$ ,  $v''' = 6$ ,  $v^{(4)} = 0$

by Leibnitz theorem

$$y^{(5)} = 4^5 e^{4x} x^3 + 5 \cdot 4^4 e^{4x} x^3 + 10(4^3) e^{4x} x^2 + 10(4^2) e^{4x} x + 5(4) e^{4x}$$

$$4^{7-3} e^{4x} x^0 + C$$