Assignment 2

1. Solution

$y = e^{x^2+x}$

$y = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x}(2x+1) + \cancel{2(e^{x^2+x})}\ 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^3+x} + 2e^{x^2+x}$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$ //

From the above equation,

Part A,

$A = y''$, $A' = y'''$, $A^n = y^{2+n}$

Part B,

$B = y'(2x+1)$

$u = y'$, $u^n = y^{n+1}$

$v = 2x+1$

$v' = 2$

$v''' = 0$

$\therefore B^n = (y^{n+1})(2x+1) + n(y^n)(2) + 0$

$B^n = (2x+1)y^{n+1} + 2ny^n$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x}(2x+1) + \cancel{2(e^{x^2}+x} \; 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^3+x} + 2e^{x^2+x}$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y_{11}$

From the above equation,

Part A,

$A = y''$, $A' = y'''$, $A^n = y^{2+n}$

Part B,

$B = y'(2x+1)$

$u = y'$, $u^n = y^{n+1}$

$v = 2x+1$

$v' = 2$

$v''' = 0$

$\therefore A^n = B^n + C^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$

$\therefore y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2. Solution

(i) $y = x^3 e^{4x}$, $y^{(5)}$

Let $u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u^n = 4^n e^{4x}$

Let $v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v'''' = 0$

By Leibniz theorem,

$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} \cdot 4^{n-2} e^{4x} \cdot 6x +$

$\frac{n(n-1)(n-2)}{3!} \cdot 4^{n-3} e^{4x} \cdot 6 + 0$

$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 n \cdot 4^{n-1} e^{4x} + 3n(n-1) \cdot 4^{n-2} e^{4x} x +$
$n(n-1)(n-2) \cdot 4^{n-3} e^{4x}$

$\therefore y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2(5) \cdot 4^4 e^{4x} + 3(5)(4) \cdot 4^3 e^{4x} \cdot x +$
$(5)(4)(3) \cdot 4^2 e^{4x}$

$y^5 = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} \cdot x + 960 e^{4x}$

$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$