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18/Eng 04/080
Electrical/Electronics Engineering
ENG 381

Assignment 2:

1.) $y = e^{x^2+x}$

Show that $y'' = y'(2x+1) + 2y$ & hence, prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln:

$y = e^{x^2+2}$

$y^{(1)} = (2x+1)e^{x^2+x}$

$y^{(11)} = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x} \cdot 2$

$\therefore y^{(11)} = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$\Rightarrow y^{(11)} = (2x+1)y^{(1)} + 2y$

finding the nth derivative;

for $y^{(11)} = (2x+1)y^{(1)} + 2y$, let's say $w_1 = (2x+1)y^{(1)}$ & $w_2 = 2y$.

$\therefore \Rightarrow w_1 = (2x+1)y^{(1)}$

$u = y'$ $\quad V = 2x+1$

$u^n = y^{(n+1)}$ $\quad V' = 2$

$u^{(n-1)} = y^n$ $\quad V'' = 0$

$\Rightarrow w_2 = 2y$

$u = y$ $\quad v = 2$

$u^n = y^n$ $\quad v' = 0$

Substituting;

$y^{(n)} = u^{(n)}v + nu^{(n-1)}v^{(1)} + \frac{n(n-1)}{2!}u^{(n-2)}v^{(2)} + \frac{n(n-1)(n-2)}{3!}u^{(n-3)}v^{3}$

for w_1; $\Rightarrow y^{(n+1)}(2x+1) + ny^n \cdot 2$

$w_2 \Rightarrow y^n \cdot 2$

$\therefore \Rightarrow y^{(n+1)}(2x+1) + 2ny^{(n)} + 2y^{(n)}$

$\Rightarrow y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$

Therefore, we've shown that,

$y'' = y'(2x+1) + 2y$

& $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2) i) $y = x^3 e^{4x}$, determine $y^{(5)}$

$y^{(n)} = u^n v + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}}{2!}v^{(11)} + n(n-1)\ldots$

$\ldots\frac{(n-2)}{3!}u^{(n-3)}v^{(111)} + \frac{n(n-1)(n-2)(n-3)u^{(n-4)}}{4!}v^{(4)} + \ldots$

let $u = e^{4x}$ $\quad v = x^3$

$u^n = 4^n e^{4x}$ $\quad v^{(1)} = 3x^2$

$u^{(n-1)} = 4^{(n-1)}e^{4x}$ $\quad v^{(11)} = 6x$

$u^{(n-2)} = 4^{(n-2)}e^{4x}$ $\quad v^{(111)} = 6$

$u^{(n-3)} = 4^{(n-3)}e^{4x}$ $\quad v^{(iv)} = 0$

$u^{(n-4)} = 4^{(n-4)}e^{4x}$ $\quad v^{(v)} = 0$

Substituting;

$y^{(n)} = 4^n e^{4x} \cdot x^3 + n[4^{(n-1)}e^{4x} \cdot 3x^2] + \frac{n(n-1)}{2!} \cdot 4^{(n-2)}e^{4x} \cdot 6x + \ldots +$

$+ \ldots \frac{n(n-1)(n-2)}{3!} \cdot 4^{(n-3)}e^{4x} \cdot 6 + \frac{n(n-1)(n-2)(n-3)}{4!} \ldots$

$\cdot 4^{(n-4)}e^{4x} \cdot 0 + 0$

$\Rightarrow y^{(n)} = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)}e^{4x} + \frac{n(n-1)^3}{3!}6x \cdot 4^{(n-2)}e^{4x} + \ldots$

$\ldots + n(n-1)(n-2)4^{(n-3)}e^{4x} + 0$

Recall

$y^{n} = y^{5}$, $n = 5$

Substituting $n = 5$;

$y^{(5)} = x^3 4^{(5)} e^{4x} + 3(5) x^2 4^{(5-1)} e^{4x} + 3(5)x(5-1) 4^{(5-2)} e^{4x} + 5(5-1)(5-2) 4^{(5-3)} e^{4x} + 0$

$y^{(5)} = 1024x^3 e^{4x} + 15(256) x^2 e^{4x} + 15(256) x e^{4x} + 60(16) e^{4x} + 0$

$y^{(5)} = 1024x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

2) ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

Soln: recall $\frac{d^2y}{dx^2} = y^{(11)}$; $\frac{dy}{dx} = y^{(1)}$

$x^2 y^{(11)} + x y^{(1)} + y = 0$

let's make $w_1 = x^2 y^{(11)}$, $w_2 = x y^{(1)}$ & $w_3 = y$

from $w_1 = x^2 y^{(11)}$

$u = y^{11}$ $\quad$ $v = x^2$

$u^n = y^{(n+2)}$ $\quad$ $v' = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\quad$ $v'' = x$

$u^{(n-2)} = y^{n}$ $\quad$ $v''' = 0$

$w_2 = x y^{(1)}$

$u = y^{(1)}$ $\quad$ $v = x$

$u^n = y^{(n+1)}$ $\quad$ $v' = 1$

$u^{(n-1)} = y^{n}$ $\quad$ $v'' = 0$

$w_3 = y$

$u = y \qquad v = 1$

$u^n = y^n \qquad v' = 0$

Substituting;

$y^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \frac{n(n-1)(n-2)}{3!} \cdots$

$\cdots u^{(n-3)}v''' + n(n-1)(n-2)(n-3)u^{(n-1)}v^{(1)} + \cdots$

$w_1 \Rightarrow y^{(n+2)} \cdot x^2 + ny^{(n+1)} \cdot 2x + \frac{n(n-1)}{2!}y^{(n)} \cdot 2 + 0$

$w_2 \Rightarrow y^{(n+1)} \cdot x + ny^n \cdot 1 + 0$

$w_3 \Rightarrow y^n + 0$

$\Rightarrow x^2y^{(n+2)} + 2xny^{(n+1)} + ny^{(n)}(n-1) + 2xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$

$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + n(n-1)y^{(n)} + ny^{(n)} + y^{(n)} = 0$

$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}[n^2 - n + n + 1] = 0$

$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + (n^2+1)y^n = 0$

$\therefore$ we've proved that;

$\Rightarrow x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$