Olomowewe Rashida (17/ENG04/057) (Electrical Electronics Engineering)

OLOMOWEWE RASHIDA OMOWUNMI
17/ENG04/057
ELETRICAL ELETRONICS ENGINEERING
ENGINEERING MATH ASSIGNMENT

1) If $y = e^{x^2+x} \quad \ldots (i)$

$\therefore y' = (2x+1)e^{x^2+x} \quad \ldots (ii)$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2 \cdot e^{x^2+x}$

Considering equation (i) and (ii)

$\therefore y'' = (2x+1)y' + 2 \cdot y$

$\therefore$ From the solution above, it has been proven that;

$y'' = (2x+1)y' + 2 \cdot y$

2) Prove that $y^{(n+2)} = (2x+1)\, y^{(n+1)} + 2(n+1)y^n$

From; $y'' = (2x+1)(y') + 2y$

Let $w_1 = y''$

$w_2 = (2x+1)(y')$

$w_3 = 2y$

From; $w_1 = y''$

$\therefore u = y'' \qquad v = 1$

$u^n = y^{n+2} \qquad v' = 0$

$\therefore W_2 = (2x+1)(y')$

$u = y' \quad , \quad v = 2x+1$

$u^n = y^{n+1} \qquad v' = 2$

$u^{n-1} = y^n \qquad v'' = 0$

$W_3 = 2y$

$u^n = y^n \qquad v = 2$

$u^n = y^n \qquad v' = 0$

$\therefore w_1 + w_2 + w_3$

Using The Formula; $u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2$

$w_1 \Rightarrow y^{n+2} \cdot 1$

$w_2 \Rightarrow y^{(n+1)}(2x+1) + n y^n \cdot 2$

$\Rightarrow y^{(n+1)}(2x+1) + 2n y^n$

$w_3 \Rightarrow y^n \cdot 2$

$\therefore w_1 + w_2 + w_3$

$\Rightarrow y^{n+2} = y^{(n+1)}(2x+1) + 2n y^n + 2y^n$

$y^{n+2} = y^{(n+1)}(2x+1) + y^n(2n+2)$

$y^{n+2} = y^{n+1} \cdot (2x+1) + 2(n+1)\, y^n$

Q.E.D.

Question 2.

(5) $y = x^3 e^{4x}$ Determine $y^{(3)}$

$\therefore u = e^{4x} \qquad v = x^3$

$\therefore u^n = 4^n e^{4x} \qquad v' = 3x^2$

$u^{n-1} = 4^{(n-1)} e^{4x} \qquad v'' = 6x$

$u^{n-2} = 4^{(n-2)} e^{4x} \qquad v''' = 6$

$u^{n-3} = 4^{(n-3)} e^{4x} \qquad v^{iv} = 0$

$\therefore$ Using Leibnitz formula;

$y^n \Rightarrow u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^4$

$\therefore y^n \Rightarrow 4^n e^{4x} \cdot x^3 + n(4^{(n-1)} e^{4x}) \cdot 3x^2 + \frac{n(n-1)}{2!}(4^{(n-2)} e^{4x}) \cdot 6x + \frac{n(n-1)(n-2)}{3!}(4^{(n-3)} e^{4x}) \cdot 6 + 0$

$y^n \Rightarrow 4^n e^{4x} \cdot x^3 + n(4^{(n-1)} e^{4x}) \cdot 3x^2 + n(n-1)(4^{(n-2)} e^{4x}) \cdot 3x +$
$n(n-1)(n-2)(4^{(n-3)} e^{4x})$.

$\Rightarrow y^n = 4^n e^{4x} \cdot x^3 + 3x^2 \cdot n\left(4^{(n-1)} e^{4x}\right) + 3x\left(n(n-1)\left(4^{(n-2)} e^{4x}\right)\right) +$
$n(n-1)(n-2)\left(4^{(n-3)} e^{4x}\right).$

$\therefore y^{(5)} = x^3 4^5 e^{4x} + 3x^2 \cdot 5\left(4^{(5-1)} e^{4x}\right) + 3x\left(5(5-1)\right) 4^{(5-2)} e^{4x} +$
$5(5-1)(5-2)\left(4^{(5-3)} e^{4x}\right).$

$y^5 = 1024x^3 e^{4x} + 3840x^3 e^{4x} + 3840x e^{4x} + 960 e^{4x}.$

(2) $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

Show that; $x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^n = 0.$

$\therefore$ From; $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0.$

$\therefore \underbrace{x^2 y''}_{w_1} + \underbrace{x y'}_{w_2} + \underbrace{y}_{w_3} = 0.$

$\therefore w_1 = x^2 y''$
$w_2 = x y'$
$w_3 = y$

$\therefore$ From; $w_1 = x^2 y''$

$u = y''$, $v = x^2.$

$u^n = y^{n+2}$ $\quad v' = 2x$

$u^{n-1} = y^{n+1}$ $\quad v'' = 2$

$u^{n-2} = y^n$ $\quad v''' = 0.$

From; $w_2 = x y'$; $u = y'$, $v = x.$

$u^n = y^{n+1}$ $\quad v' = 1.$

From; $w_3 = y$; $u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0.$

Using Formula;

$= u^n v + n u^{(n-1)} v' + \dfrac{n(n-1)}{2!}\left(u^{n-2}\right) v'' + \dfrac{n(n-1)(n-2)}{3!}\left(u^{n-3}\right) v'''$

$W_1^{(n)} = y^{(n+2)} \cdot x^2 + n y^{(n+1)} \cdot 2x + \dfrac{n(n-1)}{2!} \cdot y^{(n)} \cdot 2 + 0$

$W_2^{(n)} = y^{(n+1)} \cdot x + n(y^n) \cdot 1 + 0.$

$W_3 = y^n \cdot 1.$

$\therefore w_1 + w_2 + w_3.$

$\Rightarrow y^{n+2} \cdot x^2 + y^{(n+1)}(2xn + x) + y^{(n+2)} x^2 + y^{(n+1)} x(2n+1) + y^n(n(n-1) + n + 1)$

$\Rightarrow y^{n+2} \cdot x^2 + y^{(n+1)} x(2n+1) + y^n(n^2 - n + n + 1) = 0$

$\Rightarrow y^{n+2} \cdot x^2 + y^{(n+1)} x(2n+1) + y^n(n^2+1) = 0.$

$\therefore$ From the solution it has been shown that;

$\Rightarrow x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^n = 0.$