Durmaye Aheem O.
17/Ene03/016

Assignment 2

1 If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)\,y^{n+1} + 2y^{n}(n+1)$

$y = e^{x^2+x}$ ——— (1)

Using chain rule let $u = x^2 + x \therefore y = e^u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$\frac{du}{dx} = 2x+1$ , $\frac{dy}{du} = e^u$

$$\frac{dy}{dx} = (2x+1)\,e^{x^2+x} = y'$$

To find $y''$ we use product rule

$y' = (2x+1) = u \times, (e^{x^2+x}) = v$

$y' = uv$

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$du = 2$

$= dv = (2x+1)\,e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + (e^{x^2+x})\,2$

Recall $(2x+1)e^{x^2+x} = y'$ , $e^{x^2+x} = y$

$y'' = (2x+1)y' + 2y$ — — — (11)

To prove $y^{n+2} = (2x+1)\, y^{n+1} + 2(n+1)\, y^{n}$ --- (iii)

$(2x+1)\, y^{(n+1)} + 2(n+1)\, y^{n} - y^{(n+2)} = 0$ ---

Relations eqn (iii) and eqn 1

$y^{n+1} = y^{1}, \quad y^{n+2} = y^{11}$

let $w_1 = y^{1}(2x+1)$

~~let $w^{1} = v$~~

$v = 2x+1 \qquad u = y^{1}$

$v^{1} = 2 \qquad u^{n} = y^{n+1}$

$u^{n-1} = y^{n}$

Applying Leibnitz

$y^{n} = u^{n}v^{0} + n u^{n-1} v^{1} + 0$

$y^{n} = y^{n+1}(2x+1) + n y^{n}(2) + 0$

$y^{n} = (2x+1)\, y^{(n+1)} + 2n y^{n}$

let $w_2 = 2y \qquad v = 2$

$u = y \qquad v^{1} = 0$

$u^{n} = y^{n}$

$y^{n} = y^{2}(2) + 0 = 2y^{n}$

let $w_3 = \cancel{y} - y^{11} = -y^{(2)}$

$u = y^{2} \qquad v = -1$

$u_n = y_{n+2} \qquad v^{1} = 0$

$y^{n} = y^{n+2}(-1) + 0 = + y^{n+2}$

$y^{n+1}(2x+1) + 2ny^{n} + 2y_n - y^{n+2} = 0$

$y^{n+1}(2x+1) + 2y^{n}(n+1) - y^{n+2} = 0$

$\ldots (2x+1) \ldots 2\, n(n+1) \ldots y^{n+2}$

Scanned by CamScanner

2) $y = x^3 e^{4x}$

using Leibnitz

$u = e^{4x}$ , $u' = 4e^{4x}$ , $u'' = 16e^{4x}$, $u''' = 64e^{4x}$ $u^4 = 256e^{4x}$

$v = x^3$ , $v' = 3x^2$ , $v'' = 6x$ $v''' = 0$ $v^4 = 0$

$u^n = 4^n e^{4x}$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 +$

$\Rightarrow 4^n e^{4x}(x^3) + n4^{n-1} e^{4x} (3x^2) + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} 6x +$

$\frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} 6 + 0$

When $n = 5$

$\Rightarrow 4^5 e^{4x} x^3 \cdot e^{4x} + 5(4^4 e^{4x}) 3x^2 + \frac{5(5-1)}{2!} 4^3 e^{4x} 6x +$

$\frac{5(5-1)(5-2)}{3!} 4^2 e^{4x} 6 + 0$

$\Rightarrow 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840^{4x} x + 960 e^{4x}$

b) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$\downarrow \quad \downarrow \quad \downarrow$

$w^1 \quad w^2 \quad w^3$

$w^1 = x^2y''$

$v = x^2 \quad v' = 2x \quad v'' = 2 \quad v''' = 0$

$u = y'' \quad u' = y''' \quad u'' = y^{iv} \quad u''' = y^v$

$u^n = y^{n+2}$

Using leibnitz

$w_1 = U_nV^0 + nu^{n-1}v' + \frac{n(n-1)u^{(n-2)}v^2}{2!} + 0$

$= \cancel{y^{(n+2)}x^2 + ny^{n+1}}$

$w_1 \quad y^{n+2}x^2 + ny^{n+1}(2x) + \frac{n(n-1)y^n(2)}{2!} + 0$

$w_1 = x^2y^{n+2} + 2nxy^{n-1} + n^2 - ny^n$

let $w^2 = xy'$

$v = 1 \quad u' = y'' \quad u^n = y^{n+1}$

$v^2 = 0 \quad u'' = y''' \quad u^{n-1} = y^{n+1-1} = y^n$

$u^{n-2} = y^{n+1-2} = y^{n-1}$

$w_2 = u^nv + nu^{n-1}v' + 0$

$= y^{n+1}x + ny^n(1)$

$= xy^{n+1} + ny^n$

$w^3 = y$

$w^3 = u^nv + 0$

$= y^n$

$w^1 + w^2 + w^3$

$x^2y^{n+2} + 2nxy^{n+1} + xy^{n+1} + (n^2-n)y^n + ny^n + y^n = 0$

$\therefore x^2y^{n+2} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Scanned by CamScanner