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17/Eng04/041
ENG 381 assignment II

If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$.
$y^{(n+2)} = (2x+1)y^{n+1} + 2y^n(n+1)$

$y = e^{x^2+x}$ — (i)

using chain rule
let $u = x^2+x$, $du/dx = 2x+1$
$y = e^u$, $dy/du = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $e^u \times 2x+1$

$= (e^{x^2+x})\, 2x+1$

$y' = (e^{x^2+x})\, 2x+1$ — (ii)

To find $y''$
$y' = (e^{x^2+x})\, 2x+1$
using product rule
$u = e^{x^2+x}$ $2x+1$
$du/dx = 2$
$v = e^{x^2+x}$ $dv/dx = (e^{x^2+x})\, 2x+1$

$y'' = \frac{Vdu}{dx} + \frac{udv}{dx} \Rightarrow \frac{Vdu}{dx} + \frac{udv}{dx}$

$y'' = ((e^{x^2+x}) \times 2) + (2x+1)(e^{x^2+x})(2x+1)$ — (iii)
recall $y' = (2x+1)(e^{x^2+x})$ — (iv)
$y = e^{x^2+x}$ — (v)
Substitute in eq (i) and (iv) into (iii)
$y'' = 2(y) + (y')(2x+1)$
$y'' = 2y + (2x+1)y'$ — (v)

prove $y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$ — (v)

$(2x+1)y^{(n+1)} + 2(n+1)y^n - y^{(n+2)} = 0$

Relating eqn (v) and I

$y^{n+1} = y'$, $y^{n+2} = y''$

(c) $\omega_1 = y'(2x+1)$

$v = 2x+1$

$v' = 2$

$u = y'$

$u^n = y^{n+1}$

$u^{n-1} = y^n$

Applying leibnitz theorem

$y^n = u^n v^0 + n u^{n-1} v' + 0$

$y^n = y^{n+1}(2x+1) + n y^n (2) + 0$

$y^n = (2x+1)y^{(n+1)} + 2ny^n$

$\omega_2 = 2y$

$u = y$ $\quad v = 2$

$u^n = y^n$ $\quad v' = 0$

$y^n = y^n(2) + 0$

(i) $\omega_3 = -y'' = -y^{(2)}$

$u = y^2$ $\quad v = -1$

$u^n = y^{n+2}$ $\quad v' = 0$

$y^n = y^{n+2}(-1) + 0 = -y^{n+2}$

$y^{n+1}(2x+1) + 2ny^n + 2y^n - y^{n+2} = 0$

$y^{n+1}(2x+1) + 2y^n(n+1) - y^{n+2} = 0$

$y^{n+1}(2x+1) + 2y^n(n+1) = y^{n+2}$

2) $y = x^3 e^{4x}$

Using leibnitz theorem

$u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u''' = 64e^{4x}$, $u^{iv} = 256e^{4x}$

$v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v^{iv} = 0$

$u^n = 4^n e^{4x}$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + 0$

$\Rightarrow 4^n e^{4x}(x^3) + n4^{n-1}e^{4x}(3x^2) + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} 6x +$

$\frac{n(n-1)(n-2)}{3!} 4^{n-3} 4x \cdot 6 + 0$

When $n = 5$

$\Rightarrow 4^5 e^{4x} x^3 e^{4x} + 5(4^4 e^{4x})3x^2 + \frac{5(5-1)}{2!} 4^3 e^{4x} 6x +$

$\frac{5(5-1)(5-2)4^2 e^{4x} 6}{3!} + 0$

$\Rightarrow 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840^{4x} + 960 e^{4x}$

5) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2 y'' + x y' + y = 0$

$w_1 \quad w_2 \quad w_3$

$W_1 = x^2 y''$

$V = x^2, \quad V' = 2x, \quad V'' = 2 \quad V''' = 0$

$U = y'' \quad U' = y''' \quad U'' = y^{iv} \quad y''' = y^v$

$U^n = y^{n+2}$

Using leibnitz

$W_1 = U_n V^0 + nU^{n-1}V' + \frac{n(n-1)U^{(n-2)}}{2!} V^2 + 0$

$W_1 = y^{n+2} x^2 + ny^{n+1}(2x) + \frac{n(n-1)(y^n)(2)}{2!} + 0$

$W_1 = x^2 y^{n+2} + 2nxy^{n+1} + n^2 - ny^n$

$W_2 = xy'$

$V' = 1 \quad U' = y'' \quad U^n = y^{n+1}$

$V^2 = 0 \quad U'' = y''' \quad U^{n-1} = y^{n+1-1} = y^n$

$U^{n-2} = y^{n+1-2} = y^{n-1}$

$W_2 = U^n V + nU^{n-1}V' + 0$

$= y^{n+1}x + ny^n(1)$

$= xy^{n+1} + ny^n$

$W_3 = y \quad W^3 = U^n V + 0$

$= y^n$

$W' + W_2 + W_3$

$x^2 y^{n+2} + 2nxy^{n+1} + xy^{n+1} + (n^2 - n)y^n + ny^n + y^n = 0$

$= x^2 y^{n+2} + (2n+1)xy^{(n+1)} + (\cancel{n^2}\ (n^2+1)) y^n = 0$