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CIVIL ENGINEERING

17/ENG03/038

ENG 381 − ASS 2

① If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$. And hence prove that:

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$.

SOL

$y = e^{x^2+x}$ —(i) $\frac{dy}{dx} = y' = (2x+1)\cdot e^{x^2+x}$ —(ii)

$\frac{d^2y}{dx^2} = y'' = \left[(2x+1)\cdot(2x+1)e^{x^2+x}\right] + e^{x^2+x}\cdot 2$ —(iii)

sub equ (i) and (ii) into equ (iii) we have:

$y'' = (2x+1)y' + 2y$

If $\omega_1 = y''$

$v = 1$, $v^{(1)} = 0$

$u = y''$, $u' = y'''$ ∴ $u^n = y^{n+2}$

$\omega_1^n = u^n v + n u^{n-1} v^{(1)}$

$\omega_1^n = y^{n+2}\cdot 1 + n u^{n+1}\cdot 0 = y^{n+2}$

If $\omega_2 = (2x+1)y'$ ∴ $v = 2x+1$, $v^{(1)} = 2$, $v^{(2)} = 0$

$u = y'$ ∴ $u^n = y^{n+1}$

$\omega_2^n = u^n v + n u^{n-1} v^{(1)} + \frac{n(n-1)}{2} u^{n-2} v^{(2)}$

$= y^{n+1}\cdot(2x+1) + n y^{n}\cdot 2 + \frac{n(n-1)}{2} y^{n-1}\cdot 0$

$\omega_2^n = (2x+1)y^{n+1} + 2n y^n$

If $\omega_3 = 2y$ ∴ $v = 2$, $v^{(1)} = 0$

$u = y$, $u^n = y^n$

$\omega_3^n = u^n v + n u^{n-1} v^{(1)}$

$= y^n\cdot 2 + n y^{n-1}\cdot 0 = 2y^n$

∴ $\omega_1^n = \omega_2^n + \omega_3^n$

$y^{n+2} = (2x+1)y^{n+1} + 2n y^n + 2y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$ //

② Using Leibnitz theorem, given that

(i) $y = x^3 e^{4x}$, determine $y^{(5)}$

SOL

$y = x^3 e^{4x}$ $\therefore$ $V = x^3,\ V^{(1)} = 3x^2,\ V^{(2)} = 6x,\ V^{(3)} = 6,\ V^{(4)} = 0$

$U = e^{4x},\ U^{(1)} = 4e^{4x},\ U^{(2)} = 16e^{4x},\ U^{3} = 64e^{4x}$

$\therefore U^n = 4^n e^{4x}$

$$y^n = U^n V + nU^{n-1}V^{(1)} + \frac{n(n-1)}{2}U^{n-2}V^{(2)} + \frac{n(n-1)(n-2)}{3\cdot 2}U^{n-3}V^{(3)} + \frac{n(n-1)(n-2)(n-3)}{4!}U^{n-4}V^{(4)}.$$

$$y^n = 4^n e^{4x}\cdot x^3 + n4^{n-1}e^{4x}\cdot 3x^2 + \frac{n(n-1)}{2}4^{n-2}e^{4x}\cdot 6x + \frac{n(n-1)(n-2)}{3\cdot 2}4^{n-3}e^{4x}\cdot 6 + \frac{n(n-1)(n-2)(n-3)}{4!}4^{n-4}e^{4x}\cdot 0.$$

$$y^n = x^3 4^n e^{4x} + n3x^2 4^{n-1}e^{4x} + n(n-1)4^{n-2}e^{4x}\cdot 3x + n(n-1)(n-2)4^{n-3}e^{4x}$$

$$y^n = 4^{n-3}e^{4x}\left[4^3x^3 + n4^2 3x^2 + n(n-1)4\cdot 3x + n(n-1)(n-2)\right]$$

$$\therefore y^{(5)} = 4^{5-3}e^{4x}\left[4^3x^3 + 5\cdot 4^2\cdot 3x^2 + 5(5-1)4\cdot 3x + 5(5-1)(5-2)\right]$$

$$y^{(5)} = 16e^{4x}\left[64x^3 + 240x^2 + 240x + 60\right]$$

(ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

SOL

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0 \Rightarrow x^2y'' + xy' + y = 0$

If $\omega_1 = x^2y''$ $\therefore$ $V = x^2,\ V^{(1)} = 2x,\ V^{(2)} = 2,\ V^{(3)} = 0$

$U = y'',\ U^{(1)} = y'''$ $\therefore U^n = y^{n+2}$

$$\omega_1^n = U^nV + nU^{n-1}V^{(1)} + \frac{n(n-1)}{2}U^{n-2}V^{(2)} + \frac{n(n-1)(n-2)}{3\times 2}U^{n-3}V^{(3)}$$

$$= y^{n+2}\cdot x^2 + ny^{n+1}\cdot 2x + \frac{n(n-1)}{2}y^n\cdot 2 + \frac{n(n-1)(n-2)}{3\times 2}y^{n-1}\cdot 0$$

$$= x^2y^{n+2} + 2xny^{n+1} + n(n-1)y^n$$

If $\omega_2 = xy'$ $\therefore$ $v = x$, $v^{(1)} = 1$, $v^{(2)} = 0$

$u = y'$, $u^{(1)} = y''$ $\therefore$ $u^n = y^{n+1}$

$$\omega_2^n = u^n v + n u^{n-1} v^{(1)} + \frac{n(n-1)}{2} u^{n-2} v^{(2)}$$

$$= y^{n+1} \cdot x + n y^n \cdot 1 + \frac{n(n-1)}{2} y^{n-1} \cdot 0$$

$$= x y^{n+1} + n y^n$$

If $\omega_3 = y$ $\therefore$ $v = 1$ $v^{(1)} = 0$

$u = y$, $u^n = y^n$

$$\omega_3^n = u^n v + n u^{n-1} v^{(1)}$$

$$= y^n \cdot 1 + n y^{n-1} \cdot 0 = y^n$$

$\therefore$ $\omega_1^n + \omega_2^n + \omega_3^n = 0$

$$\Rightarrow x^2 y^{n+2} + 2xn y^{n+1} + n(n-1) y^n + x y^{n+1} + n y^n + y^n$$

$$= x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2 - n + n + 1) y^n$$

$$= x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$$