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17/ENG04/038
Elect/Elect.

ENG381 Assignment 2.

If $y = e^{x^2+x}$.

Show that $y'' = y'(2x+1)2y$
and hence prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + x(n+1)y^{n}$

Soln

$y = e^{x^2+x}$

Differentiating using function of function

$u = x^2+x \qquad y = e^u$

$\frac{du}{dx} = 2x+1 \qquad \frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

$\frac{dy}{dx} = (2x+1) \cdot e^u$

$y' = e^{(x^2+x)} \cdot (2x+1)$

But $y' = \frac{dy}{dx}$

$y' = e^{(x^2+x)} \cdot (2x+1)$

Differentiating Using product rule

$y'' = \frac{d}{dx}\left[(2x+1)e^{x^2+x}\right] = \frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = u\frac{dv}{dx} + v\frac{du}{dx}$

Let $u = 2x+1$

$\frac{du}{dx} = 2$

$v = e^{x^2+x}$

Substituting them into product formula

$y'' = v\frac{du}{dx} + u\frac{dv}{dx}$

$y'' = e^{x^2+x} \cdot 2 + 2x+1 \cdot (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + 2x+1 \cdot (2x+1)e^{x^2+x}$

But $y = e^{x^2+x}$; $y' = (2x+1)e^{x^2+x}$

$y'' = y'(2x+1) + 2y$ —— $(*)$

from eqn $(*)$ above

$y'(2x+1) + 2y - y'' = 0$

let $w_1 = y'(2x+1)$

$w_2 = 2y$

$w_3 = -y''$

$W_1 = y'(2x+1)$

$u = y'$ $\quad v = 2x+1$

$u^n = y^{n+1}$ $\quad v' = 2$

$u^{n-1} \cdot y^n$ $\quad v^2 = 0$

Using Lebnitz theorem:

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \cdots$$

Substituting:

$y^n = y^{n+1} \cdot 2x+1 + n \cdot y^n \cdot 2 \cancel{+} 0$

$y^n = y^{(n+1)} \cdot (2x+1) + 2n y^n$

$W_2 = 2y$

$u = y$ $\quad v = 2$

$u^n = y^n$ $\quad v_2' = 0$

Using Lebnitz theorem

$y^n = y^n \cdot 2$; $y^n = 2y^n$

$W_3 = -y^2$

$u = y^2$ $\quad v = -1$

$u^n = y^{n+2}$ $\quad v = 0$

Using lebnitz therem

$y^n = y^{n+2} \cdot -1$

$y^n = -y^{n+2}$.

Summing All togethere:

$$y^n = y^{(n+1)} \cdot 2x+1 + 2ny^n + 2y^n - y^{n+2} = 0$$

Collecting like term

$$y^{n+2} = y^{n+1} \cdot (2x+1) + 2y^n(n+1) = 0$$

$$y^{n+2} = y^{n+1}(2x+1) + 2(n+1)y^n = 0$$

Question 2:

Using Leibnitz theorem, given that

$y = x^3 e^{4x}$

soln

$y = x^3 \cdot e^{4x}$

$U = e^{4x}$ $\quad V = x^3$

$U^n = 4^n e^{4x}$ $\quad V' = 3x^2$

$U^{n-1} = 4^{(n-1)} e^{4x}$ $\quad V^2 = 6x$

$U^{n-2} = 4^{(n-2)} e^{4x}$ $\quad V^3 = 6$

$U^{n-3} = 4^{(n-3)} e^{4x}$ $\quad V^4 = 0$

Using Leibnitz theorem

$$y^n = U^n v + nU^{n-1}v' + \frac{n(n-1)}{2!} U^{n-2} V^2 + \frac{n(n-1)(n-2)}{3!} U^{n-3} V^3 + \dots$$

$$\cancel{y^n = 4^n e^{4x} \cdot x^3 + 4^{(n-1)} e^{4x} \cdot 3x^2 + n^2 - n \cdot 4^{n-2} e^{4x} \cdot 3x + n(n^2-3n+2) \cdot 4}$$

$$y^n = 4^n e^{4x} \cdot x^3 + n4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{(n)(n-1)}{2!} 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} 4x \cdot 6 + 0$$

$$y^n = 4^n e^{4x} \cdot x^3 + n4^{(n-1)} e^{4x} \cdot 3x^2 + n^2 - n \cdot 4^{n-2} e^{4x} \cdot 3x + n(n^2-3n+2) \cdot 4^{n-3} e^{4x} \cdot 2$$

factorizing e^{4x}

$$y^n = e^{4x}\left[(4^n \cdot x^3) + (n^{4n-1} \cdot 3x^2) + (n^2 - n\, 4^{n-2} \cdot 3x) + (n^3 - 3n^2 + 2n\, 8^{n-3})\right]$$

factorizing 4^{n-3}

$$y^n = e^{4x}\left[(4^{n-n+3} \cdot x^3) + (n^{4(n-1-n+3)} \cdot 3x^2) + (n^2 - n4^{n-2-n+3} \cdot 3x) + (n^3 \cdot 3n^2 + 2n\, 8^{n-n-n+3})\right]$$

$$y^n = e^{4x}\left[(4^3 x^3) + (n^{4^2} \cdot 3x^2) + (n^2 - n \cdot 4^1 \cdot 3x) + (n^3 - 3n^2 + 2n 8^0)\right]$$

But $n = 5$

$$y^5 = e^{4x} \cdot 4^{n-3}\left[64x^3 + 48nx^2 + 12xn^2 - n + n^3 - 3n^2 + 2n\right]$$

$$y^5 = e^{4x} 4^2\left[64x^3 + 240x^2 + 240x + 60\right]$$

$$y^5 = 16e^{4x}\left[64x^3 + 240x^2 + 240x + 60\right].$$

11) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Rewriting equation :

$x^2y'' + xy' + y = 0$

for $w_1 = x^2y''$

$w_2 = xy'$

$w_3 = y$

for w_1

$u = y^2 \qquad v = x^2$

$u^n = y^{n+2} \qquad v' = \cancel{2x}$

$u^{n-1} = y^{n+1} \qquad v^2 = 2$

$u^{n-2} = y^n \qquad v^3 = 0$

for w_2

$u = y' \qquad v = x$

$u^n = y^{n+1} \qquad v' = 1$

$u^{n-1} = y^n \qquad v^2 = 0$

for w_3

$u = y \qquad v = 1$

$u^n = y^n \qquad v' = 0$

Using lebnitz theorem

$$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)}{3!}u^{n-2}v^2 + \dots$$

for w_1

$y^n = y^{(n+2)}\cdot x^2 + ny^{(n+1)}\cdot 2x + \frac{n(n-1)}{2}y^n\cdot 2 + 0$

for w_2 ; $y^n = y^{(n+1)}\cdot x + n\cdot y^n\cdot 1 + 0$

for w_3 ; $y^n = y^n\cdot 1 + 0$

Summing together .

$y^n = x^2y^{(n+2)} + ny^{(n+1)}\cdot 2x + \frac{n(n-1)}{2}y^n\cdot 2 + y^{(n+1)}\cdot x + ny^n + y^n = 0$

$y^n = x^3y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$y^n = x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2+1) = 0$

$y^n = x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$.