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17/ENG07/011
Petroleum Engineering

ENG381: Engineering Mathematics

Assignment 2

1.) Solution

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x} \cdot (2x+1) + 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

From the above equation

Part A,
$A = y''$, $A' = y'''$, $A^n = y^{2+n}$

Part B,
$B = y'(2x+1)$

$U = y'$, $U^n = y^{n+1}$
$V = 2x+1$
$V' = 2$
$V''' = 0$

$B^n = (y^{n+1})(2x+1) + an(y^n)(2) + 0$ (11)

$B^n = (2x+1)y^{n+1} + 2ny^n$

Part C

$C = 2y$

$C^n = 2y^n$

$A^n = B^n + C^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution

$y = x^3 e^{4x}, \; y^{(5)}$

let $U = e^{4x}$, $U' = 4e^{4x}$, $U'' = 16e^{4x}$, $U^n = 4^n e^{4x}$

let $V = x^3$, $V' = 3x^2$, $V'' = 6x$, $V''' = 6$, $V'''' = 0$

By Lebniz theorem

$$y^n = 4^n e^{4x} x^3 + n \cdot 4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} 6x + \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} 6 + 0$$

$$\therefore y^5 = 4^n e^{4x} \cdot x^3 + 3x^2 n \cdot 4^{n-1} e^{4x} + 3n(n-1) \cdot 4^{n-2} e^{4x} x + (5)(4)(3) \cdot 4^2 e^{4x}$$

$$y^5 = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} \cdot x + 960 e^{4x}$$

$$y^5 = 64 e^{4x}(16x^3 + 60x^2 + 60x + 15)$$

(II) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $x^2y^{(n+2)} + (2n+1)x\,y^{(n+1)} + (n^2+1)y^n = 0$

For Part A,

$A = x^2y''$

$u = y''$, $u^n = y^{n+2}$

$v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$A^n = (y^{n+2})x^2 + n(y^{n+1})\cdot 2x + \frac{n(n-1)}{2!}\cdot(y^n)\cdot 2 = 0$

$A^n = x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n$

For Part B,

$B = xy'$

$u = y'$, $u^n = y^{n+1}$

$v = x$, $v' = 1$, $v'' = 0$

$B^n = (y^{n+1})\cdot x + n(y^n)\cdot 1 + 0$

$= xy^{(n+1)} + ny^n$

For Part C,

$C = y$

$C^n = y^n$

$\therefore A^n + B^n + C^n = 0$

$= x^2y^{(n+2)} + 2xy^{(n+1)} + (n^2 - n)y^n + xy^{(n+1)}$
$+ ny^n + y^n = 0$

$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$= x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$