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17/ENG04/076
ELECT. / ELECT.
ENG 381 ASSIGNMENT 2

1.) If $y = e^{x^2+x}$, Show that $y'' = y'(2x+1)+2y$ and hence Prove that $y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$.

Soln.

$y = e^{x^2+2}$ ------ ①

using Chain rule; Let $u = x^2+x \therefore y = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$; $\frac{du}{dx} = 2x+1$

$\frac{dy}{du} = e^u = e^{x^2+x}$

$\frac{dy}{dx} = (2x+1)e^{x^2 4x} = y'$

Finding y'', using Product rule

$y' = (2x+1)(e^{x^2+x})$

Let $u = 2x+1$; $dy/dx = 2$

Let $V = e^{x^2+x}$; $dy/dx = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = y'' = u\,dv/dx + v\,du/dx$

$y'' = (2x+1)(2x+1)(e^{x^2+x}) + e^{x^2+x}(2)$

$y = e^{x^2+x}$ & $y' = (2x+1)e^{x^2+x}$

Subst. the values of y into y''

$y'' = (2x+1)(2x+1)(e^{x^2+x})$

$y'' = (2x+1)y' + 2y$ .... ②

To Prove that $y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$ .... ③

$(2x+1)y^{n+1} + 2(n+1)y^n - y^{(n+2)} = 0$

Relating eqn ③ and ②

$y^{n+1} = y', y^{n+2} = y''$

Let $w = y'(2x+1)$

$v = 2x+1$ $u = y'$

$v' = 2$ $u^n = y^{n+1}$

$u^{n-1} = y^{(n+1)-1} = y^n$

Applying Leibniz theorem

$$y^n = u^n v + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2!} v^{(2)} u^{(n-2)} + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \dots$$

$$y^n = y^{(n+1)}(2x+1) + n y^n (2) + 0$$

$$y^n = (2x+1) y^{(n+1)} + 2(n) y^n$$

Let $w_2 = 2y$

$u = y \qquad V = 2$

$u^n = y^n \qquad V' = 0$

$y^n = y^n(2) + 0 = 2y^n$

Let $w_3 = -y''$

$-y'' = -(y^{(2)})$

$u = y^2 \qquad V = -1$

$u^n = y^{n+2} \qquad V' = 0$

$y^n = y^{n+2}(-1) + 0 = -y^{n+2}$

$y^{n+1}(2x+1) + 2ny^n + 2yn - y^{n+2} = 0$

$y^{n+1}(2x+1) + 2y^n(n+1) - y^{n+2} = 0$

$\therefore \quad y^{n+1}(2x+1) + 2y^n(n+1) = y^{n+2}$ //

2.) Using the Leibniz theorem, given that (i) $y = x^3 e^{4x}$, determine y^5, (ii) $x^2 \, d^2y/dx^2 + x \, dy/dx + y = 0$, Show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

Soln.

i) $y = x^3 e^{4x}$

Using Leibnitz theorem

$u = e^{4x}, \; u' = 4e^{4x}, \; u'' = 16e^{4x}, \; u''' = 64e^{4x}, \; u^{IV} = 256e^{4x}$

$V = x^3, \; V' = 3x^2, \; V'' = 6x, \; V''' = 0$

$\therefore u^n = 4^n e^{4x}$

$$y^n = u^n v + n u^{n-1} v^1 + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 +$$

$$\frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^4 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} u^{n-5} v^5$$

$= 4^n e^{4x}(x^3) e^{4x} + n 4^{n-1} e^{4x}(3x^2) + \frac{n(n-1) 4^{n-2}}{2!} e^{4x} 6x + \frac{(n)(n-1)(n-2)}{3!}$

$4^{n-3} e^{4x} 6 + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-1}(e^{4x})(0)$

Since $n=5$

$y^5 = \cancel{4^5}\, 4^5 e^{4x} x^3 e^{4x} + 5(4^4 e^{4x}) 3x^2 + \frac{5(5-1)}{2!} 4^3 e^{4x} 6x + \frac{5(5-1)(5-2)}{3!} 4^2 e^{4x} 6$

$+ 0$

$\cancel{y^5 = 4^5 e^{4x} x^3 + 5(4^4}$

$\cancel{y^5 = 4^5 e^{4x}}$

$y^5 = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 960 e^{4x}$.

$y^5 = 64 e^{4x}(16x^3 + 60x^2 + 60x + 15)$ Ans

iii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

Let $g = x^2 y''$

$v = x^2 \qquad u^n = y'' = y^{(n+2)}$

$v' = 2x \qquad u^n = y^{n+2}$

$v'' = 2 \qquad u^{n-1} = y^{(n+2)-1} = y^{n+1}$

$v''' = 0 \qquad u^{n-2} = y^{(n+2)-2} = y^n$

Using Leibniz theorem

$g_1^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + 0$

$g_1^n = x^2 y^{n+2} + (2x)(n) y^{n+1} + n(n-1) y^n$

$g_1^n = x^2 y^{n+2} + 2nx y^{n-1} + n^2 - n y^n$

Let $g_2 = xy'$

$u = y', u^n = y^{(n+1)}$

$v' = 1 \qquad u^{n+1} = y^{n+1-1} = y^n$

$v^2 = 0 \qquad u^{n-2} = y^{(n+1)-2} = y^{n-1}$

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$g_2^n = u^n v + n u^{n-1} v' + 0$

$g_2^n = y^{(n+1)} x + n(y^n)(1)$

$g_2^n = x y^{n+1} + n y^n$

Let $g_3 = y$

$v = 1 \qquad u = y, \; u^n = y^n$

$v' = 0 \qquad u^{n-1} = y^{n-1}$

$g_3^n = \cancel{y}\, u^n v + 0$

$y^{n(1)}$

$g_3^n = y^n$

$g_1^n + g_2^n + g_3^n = 0$

$x^2 y^{n+2} + 2nx y^{n+1} + x y^{n+1} + (n^2 - n) y^n + n y^n + y^n = 0$

$\therefore \; x^2 y^{n+2} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$,,