ABDULAZEEZ ZUBEIDAT

17/ENG03/001

CIVIL ENGINEERING

ENGINEERING MATHS III

i) If $y = e^{x^2+x}$ . . . (i)

$\therefore y' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2 \cdot e^{x^2+x}$

Considering equation (i) and (ii)

$\therefore y'' = (2x+1)y' + 2 \cdot y$

From the solution above it has been proven that

$y'' = (2x+1)y' + 2 \cdot y.$

2) Prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

From $y'' = (2x+1)(y') + 2y$

Let $w_1 = y''$

$w_2 = (2x+1)(y')$

$w_3 = 2y$

From; $w_1 = y''$

$u = y''$ $v = 1$

$u^n = y^{n+2}$ $v' = 0$

$\therefore w_2 = (2x+1)(y')$

$u = y'$ $v = 2x+1$

$u^n = y^{n+1}$ $v' = 2$

$u^{n-1} = y^n$ $v'' = 0$

$w_3 = 2y$

$u^n = y^n$ $v = 2$

$u^n = y^n$ $v' = 0$

$w_1 + w_2 + w_3$

Using the formular $u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2$

$w_1 \Rightarrow y^{n+2} \cdot 1$

$w_2 \Rightarrow y^{(n+1)}(2x+1) + ny^{n} \cdot 2$

$\Rightarrow y^{(n+1)}(2x+1) + 2ny^{n}$

$w_3 \Rightarrow y^{n} \cdot 2$

$\therefore w_1 = w_2 + w_3$

$\Rightarrow y^{n+2} = y^{(n+1)} \cdot (2x+1) + 2ny^{n} + 2y^{n}$

$y^{n+2} = y^{(n+1)}(2x+1) + y^{n}(2n+2)$

$y^{n+2} = y^{n+1}(2x+1) + 2(n+1)y^{n}$

(Q.E.D)

Question 2.

(a) $y = x^3 e^{4x}$ Determine $y^{(5)}$

$\therefore u = e^{4x}$ $\quad v = x^3$

$\therefore u^{n} = 4^{n} e^{4x}$ $\quad v' = 3x^2$

$u^{n-1} = 4^{(n-1)} e^{4x}$ $\quad v'' = 6x$

$u^{n-2} = 4^{(n-2)} e^{4x}$ $\quad v''' = 6$

$u^{n-3} = 4^{(n-3)} e^{4x}$ $\quad v^{iv} = 0$

1. Using leibinitz Formular

$y^{n} \Rightarrow u^{n}v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \frac{n(n-1)(n-2)}{3!}u^{(n-3)}v''' + \frac{n(n-1)(n-2)(n-3)}{4!}u^{(n-4)}v^{iv}$

$\therefore y^{n} = 4^{n}e^{4x} \cdot x^3 + n(4^{(n-1)}e^{4x}) \cdot 3x^2 + \frac{n(n-1)}{2!}(4^{(n-2)}e^{4x}) \cdot 6x + \frac{n(n-1)}{3!}(4^{(n-3)}e^{4x}) \cdot 6 + 0$

$y^{n} \Rightarrow 4^{n}e^{4x} \cdot x^3 + n(4^{(n-1)}e^{4x}) \cdot 3x^2 + n(n-1)(4^{(n-2)}e^{4x}) \cdot 3x + n(n-1)(n-2)(4^{(n-3)}e^{4x})$

$\Rightarrow y^{n} = 4^{n}e^{4x} \cdot x^3 + 3x^2 n(4^{(n-1)}e^{4x}) + 3x(n(n-1))(4^{(n-2)}e^{4x}) + n(n-1)(n-2)(4^{(n-3)}e^{4x})$.

$y^{(5)} = x^3 4^5 e^{4x} + 3x^2 \cdot 5(4^{(5-1)})e^{4x} + 3x(5(5-1))4^{(5-2)}e^{4x} + 5(5-1)(5-2)(4^{(5-3)}e^{4x}$

$y^5 = 1024x^2e^{4x} + 3840x^3e^{4x} + 3840xe^{4x} + 960e^{4x}$

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

From $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$\therefore x^2y'' + xy' + y = 0$
$W_1 \quad W_2 \quad W_3$

$W_1 = x^2y''$

$W_2 = xy'$

$W_3 = y$

From $W_1 = x^2y''$

$u = y'' \quad u' = y''' \quad u'' = y^{iv} \quad u''' = y^v$

$v = x^2 \quad v' = 2x \quad v'' = 2 \quad v''' = 0$

From $W_2 = xy' \quad u = y' \quad v = x$

$u^n = y^{n+1} \quad v' = 1$

From $W_3 = y \quad u = y \quad v = 1$

$u^n = y^n \quad v = 0$

Using formula

$= u^nv + nu^{(n-1)}v' + \frac{n(n-1)}{2!}(u^{(n-2)})v'' + \frac{n(n-1)(n-2)}{3!}(u^{n-3})v'''$

$W_1^{(n)} = y^{(n+2)} \cdot x^2 + ny^{(n+1)} \cdot 2x + \frac{n(n-1)}{2!} \cdot y^{(n)} \cdot 2 + 0$

$W_2^{(n)} = y^{(n+1)}x + n(y^n)1 + 0$

$W_3 = y^n \cdot 1$

$\therefore W_1 + W_2 + W_3$

$\Rightarrow y^{n+2} \cdot x^2 + y^{(n+1)}(2xn+x) + y^{(n+2)}x^2 + y^{(n+1)}x(2n+1) + y^n(n(n-1)+n+1)$

$\Rightarrow y^{n+2} \cdot x^2 + y^{(n+1)}x(2n+1) + y^n(n^2-n+n+1) = 0$

$\Rightarrow y^{n+2} \cdot x^2 + y^{(n+1)}x(2n+1) + y^n(n^2+1) = 0$

$\therefore$ From the solution it has been shown that

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n$