NAME: LAWAL MOHAMMAD ZAHIR
MATRIC NO: 17/ENG04/039
DEPT: Electrical/Electronics

① $y = e^{x^2} + x$

$y^{(1)} = (2x+1)e^{x^2+x}$

$y^{(2)} = 2e^{x^2+2} + (2x+1)(2x+1)e^{x^2+x}$

$y^{n} = 2e^{x^2+2} + (2x+1)^2 e^{x^2+x}$

$y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x} + (2x+1) + 2(e^{x^2+x})$

$(2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$y'' = y'(2x+1)2y$

from the equation

Part A

$A = y'', \; A' = y''', \; A'' = y^{2+n}$

PART B

$B = y'(2x+1)$

$u = y', \; u^n = y^{n+1}$

$V = 2x+1, \; V^{(1)} = 2, \; V^{(2)} = 0$

$B = y(n+1)(2x+1) + n(y^n)(2) + 0$

$B^n = (2x+1)y^{n+1} + 2ny^n$

PART C

$C = 2y$

$C^n = 2y^n$

$A^n + B^n + C^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$

Scanned by CamScanner

$y^{n+2} = (2x+1)\,y(n+2) + 2(n+1)y^n$

2) (i) $y = x^3 e^{4x}$, y^5

Let $u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u^n = 4^n e^{4x}$

Let $v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v^{iv} = 0$

Using Leibnitz theorem.

$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4(n-2) e^{4x} \cdot 6x +$
$\frac{n(n-1)(n-2)}{3!} \cdot 4(n-3) e^{4x} \cdot 6 + 0$

$y^5 = 4^5 e^{4x} \cdot x^5 + 3x^5 (5) \cdot 4^4 e^{4x} + 3(5)(4) \cdot 4^5 e^{4x} \cdot x^4 +$
$(5)(4)(3) \cdot 4^2 e^{4x}$

$y^5 = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} + 960 e^{4x}$

$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$

(ii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ Shows that $x^2 y^{(n+2)} + (2n+1)x\, y^{(n+2)} + (n^2+1)y^n = 0$

For PART A.

$A = x^2 y''$

$u = y''$, $u^n = y^{n+2}$

$v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$A'' = (y^{n+2})x^2 + n(y^{n+2})2x + \frac{n(n-1)}{2} \cdot (y^2)2 + 0$

$A'' = x^2 y^{(n+2)} 2x + \frac{n(n-1)}{2} \cdot (y^2) 2 + 0$

$A'' = x^2 y^{(n+2)} + 2xn y^{(n+1)} + n(n-1)y^n$

For PART B,

$B = xy'$

$u = y'$, $u^n = y^{n+1}$

$v = x,\ v' = 1,\ v'' = 0$

$B'' = (y^{n+1}) \cdot x + n(y^2) \cdot 1 + 0$

$= xy^{n+1} + ny^n$

For PART C

$C = y$

$C^n = y^n$

$= A^n + B^n + C^n = 0$

$= x^2 y(n+12) + 2x\, ny^{(n+1)} + (n^2 - n) y^n$

$= x^2 y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$= x^2 y^{(n+2)} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$