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17/ENG02/049
ENG 381 assignment II

If $y = e^{x^2+x}$ Show that $y'' = y'(2x+1) + 2y$

$y^{(n+2)} = (2x+1)y^{n+1} + 2y^{n}(n+1)$

$y = e^{x^2+x}$ —— (1)

using chain rule

let $u = x^2+x$ $du/dx = 2x+1$

$y = e^{u}$ $dy/du = e^{u}$

$dy/dx = \frac{dy}{du} \times \frac{dy}{dx}$ $e^{u} \times 2x+1$

$= (e^{x^2+x})\, 2x+1$

$y = e^{x^2+x})\, 2x+1$ — (ii)

To find $y^{(1)}$

$y^{(1)} = (e^{x^2+x})\, 2x+1$

using Product rule

$u = 2x+1$

$du/dx = 2$

$v = e^{x^2+x}$ $dv/dx = (e^{x^2+x})\, 2x+1$

$y'' = \frac{Vdu}{dx} + \frac{udv}{dx} \Rightarrow \frac{Vdu}{dx} + \frac{udv}{dx}$

$y'' = (e^{x^2+x}) \times 2) + (2x+1)(e^{x^2+x})(2x+1)$ — iii

recall $y' = (2x+1)(e^{x^2+x})$ — ii

$y = e^{x^{2+1}}$ — IV

Substitute in eq (i) and (ii) in (iii)

$y'' = 2(y) + (y'')(2x+1)$

$y'' = 2y + (2x+1)\, y =$ //

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Applying leibnitz theorm

$y^n = u^n v + nu^{n-1} v' + 0$

$y^n = y^{n+1}(2x+1) + ny^n(2) + 0$

$y^n = (2x+1)\, y^{n+1} + 2ny^n$

$w_2 = 2y$

$u = y \qquad v = 2$

$u^n = y^n \qquad v' = 0$

$y^n = y^n(2) + 0$

$w_3 = -y'' = -y^{(2)}$

$u = y^2 \qquad v = 1$

$u_n = y^{n+2} \qquad v' = 0$

$y^n = (2x+1) + 2ny^n + 2y^n - y^{n+2} = 0$

$y^{n+1}(2x+1) + 2y^n(n+1) = y^{n+2} = 0$

$y^{n+1}(2x+1) = 2y^n(n+1) = y^{n+2}$

(2) $y = x^3 e^{4x}$

using leibnitz theorm

$u = e^{4x} \quad u' = 4e^{4x} \quad u'' = 16e^{4x} \quad u''' = 64e^{4x}$

$u^{iv} = 256e^{4x}$

$V = x^3 \quad V' = 3x^2 \quad V'' = 6x \quad V''' = 6 \quad V^{iv} = 0$

$u^n = 4^n e^{4x}$

$y^n = u_n^n V + nu^{n-1}v' + \frac{n(n-1)u^2 v^2}{2!}$

$+ \frac{n(n-1)(n-2)\, u^{n-3} v^3}{3!} + 0$

$\Rightarrow 4^n e^{4x}(x^3) + n\, 4^{n-1} e^{4x}(3x^2) + \frac{n(n-1)\, 4^{n-2} e^{4x}\, 6x}{2} + \frac{n(n-1)(n-2)\, 4^{n} e^{4x}\, 6}{3!}$

when n =

$\Rightarrow 4^5 e^{4x}$

$+ 8$

$\Rightarrow$

$+9$

b $\frac{x^2 d}{d}$

w_1

when $n=5$

$\Rightarrow 4^5 e^{4x} x^3 e^{4x} + 5(4^4 e^{4x}) 3x^2 + \frac{5(5-1) 4^3 e^{4x} 6x}{2!}$

$+ \frac{5(5-1)(5-2) 4^2 e^{4x} 6 + 0}{3!}$

$\Rightarrow 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x}$
$+ 960 e^{4x}$

b $\frac{x^2 d^2y}{dx^2} + \frac{x dy}{dx} + y = 0$

$\underbrace{x^2 y''}_{w_1} + \underbrace{xy'}_{w_2} + \underbrace{y}_{w_3} \rightarrow$

$w_1 = x^2 y'$

$v = x^2 \quad v' = 2x \quad v'' = 2 \quad v''' = 0$

$u = y'' \quad u' = y''' \quad u'' = y^{iv} \quad u''' y^0$

$u^n = y^{n+2}$

Using Leibnitz

$w_1 = u_n v^2 + n u^{n-1} v' + \frac{n(n-1) u^{(n-2)} v^2}{2!} + 0$

$w_1 = y^{n+2} x^2 + n y^{n+1}(2x) + \frac{n(n-1)(y^n)(2)}{2!}$

$w_1 = x^2 y^{n+2} + 2nxy^{n-1} + n^2 - ny^n$

$w_2 = xy$

$v_1 = 1 \quad u' = y'' \quad u^n = y^{n+1}$

$v^2 = 0 \quad u^{u} = y''' \quad u^{n-1} y^{n+1-1} = y^n$

$u^{n-2} = y^{n-1-2} = y^{n-}$

$w^2 = u^n v + n u^{n-1} v' + 0$

$y^{n+1} x + n y^n (1)$

$x y^{n+1} + n y^n$

$w_3 = y \quad w^3 = u^n v + 0$

$= y^n$

$w_1 + w_2 + w_3$

$x^2 y^{n+2} + 2nxy^{n+1} + xy^{n+1} + (n^2 n) y^n + ny^n$
$+ y^n = 0$