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17/eng03/006
Civil Eng

The dynamic model of a body in motion performing damped forced vibration is as equation (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t = 0, x = 0.1$ and $\frac{dx}{dt} = 0$

a) using the auxiliary equation method obtain the solution of the model in form of an expression having x as a function of t

b) with the aid of a matlab m/file program, plot the relationship between x and t for $0 \le t \le 15$ time unit using a step size of 0.01 unit and

c) write the steady-state solution of the model in form of $x = k\sin(t + \alpha)$

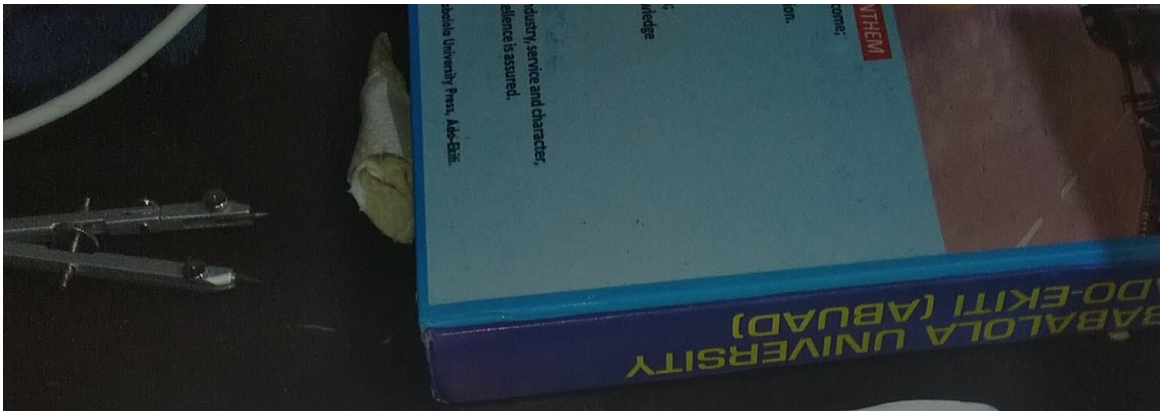
Sol

$$C.F = m^2 + 5m + 6 = 0$$
$$(m+3)(m+2) = 0$$
$$m = -3, -2$$
$$x = Ae^{-3t} + Be^{-2t}$$

$$f(t) = \cos t$$
$$x = C\cos t + D\sin t$$
$$dx/dt = -C\sin t + D\cos t$$
$$d^2x/dt^2 = -C\cos t - D\sin t$$

Substitute

$$= -C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t)$$



$\cos t(-c+5D+6c)+\sin t(-D-5c+6D)=\cos t$

$\cos t(5c+5D)+\sin t(5D-5c)=\cos t$

$5c+5D=1$

$5D-5C=0$

$10D=1$

$D=1/10$

find C

$5\left(\frac{1}{10}\right)+5c=1$

$1/2+5c=1$

$c=1/10$

G.S $= x = Ae^{-3t}+Be^{-2t}+1/10\cos t+1/10\sin t$

subs $(x=0.1)$ $t=0$

$0.1=Ae^{-0}+Be^{-0}+1/10\cos 0+1/10\sin 0$

$0.1=A+B+0.1$

$A+B=0$ ---- eqn 1

$dx/dt = -3Ae^{-3t}-2Be^{-2t}-1/10\sin t+1/10\cos t$

(a) $dx/dt=0$ $t=0$

$0=-3A-2B+1/10$

$3A+2B=1/10$

$A+B=0 \times 2$

$3A+2B=0.1$

$2A+2B=0$

$-A=-0.1$

$A=0.1$

To find B

$0.1+B=0$

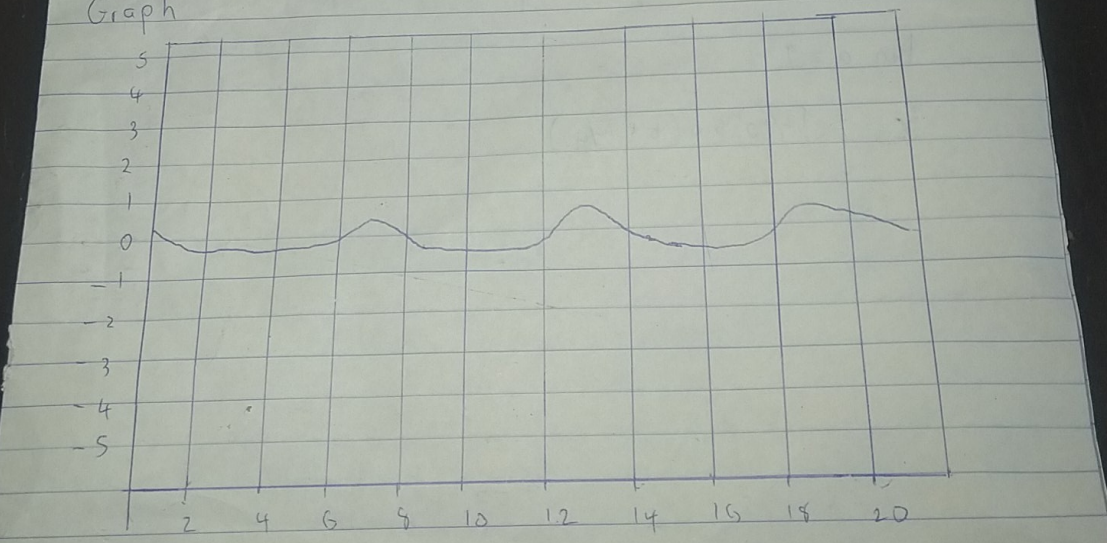
$B=-0.1$

$G.S = \frac{1}{10}e^{-3t}-\frac{1}{10}e^{-2t}+\frac{1}{10}\cos t+\frac{1}{10}\sin t$

B) MATLAB CODE

```
Command window
clear
clc
close all
syms x
syms t
t = 0:0.1:15
x = 0.1*exp(-2*t) + 0.1*exp(-3*t) + 0.1*cos(t) + 0.1*sin(t)
plot(t,x)
xlabel('t'), ylabel('x')
grid on
axis equal
```

Graph



(i) Steady state

$$x_{ss} = x_{t\to\infty} = 0.1\cos t + 0.1\sin t = K\sin(t+\alpha)$$

Recall $K\sin(t+a) = K\sin t\cos a + K\cos t\sin a$

$x_t - 2x = 0.1\cos t + 0.1\sin t = K\sin t\cos a + K\cos t\sin a$

Taking the coefficients of cos and sin

$0.1 = K\sin a$

$0.1 = H\cos a$

Square both sides

$K^2\sin^2 a + K^2\cos^2 a = \frac{1}{100} + \frac{1}{100}$

$K^2(\sin^2 a + \cos^2 a) = \frac{1}{50}$

$K^2 = 1/50 \qquad K = \sqrt{2}/10$

$\frac{K\sin a}{K\cos a} = \frac{0.1}{0.1} = 1 \quad \therefore a = \frac{\pi}{4}$

$\tan a = 1$

$x_{ss} = \sqrt{2}/10\sin(t + \pi/4)$