Orafu Franklyn
17/Engo1/025
Chemical Engineering
Eng 381(1ms2)

Question 1

1a) $y = e^{x^2+x}$

$y' = 2x+1\, e^{x^2+x}$

$y'' = 2x+1 \frac{d}{dx}(e^{x^2+x}) + e^{x^2+x}\frac{d}{dx}(2x+1)$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y \Rightarrow$ Proven

1b) $y'' - y'(2x+1) - 2y = 0$

Using leibnitz theorem

$w_1 = y''$

$w_2 = y'(2x+1)$

$w_3 = 2y$

degenerate eqn $= w_1 - w_2 - w_3 = 0$

w_1

$u = y''$ $\quad u' = y'''$ $\quad$ hence $u^n = y^{n+2}$

$v = 1$ $\quad v' = 0$

$w_1 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \ldots$

$w_1 = y^{n+2}(1) + n y^{n+1}(0)$

$w_1 = y^{n+2}$

w_2

$w_2 = y'(2x+1)$

$u = y'$ $\quad u' = y''$ $\quad u'' = y'''$, Hence $u^n = y^{n+1}$

$v = 2x+1$ $\quad v' = 2$ $\quad v'' = 0$

$w_2 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \ldots$

$w_2 = y^{n+1}(2x+1) + n y^n (2) + \frac{n(n-1)}{2!} y^{n-2}(0) + \ldots$

$\omega_2 = y^{n+1}(2x+1) + 2ny^n + 0$
$\omega_2 = y^{n+1}(2x+1) + 2ny^n$ ------ (ii)

ω_3

$u = y$ $\quad u' = y'$ $\quad$ Hence $u^n = y^n$
$v = 2$ $\quad v' = 0$

$\omega_3 = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \ldots$

$\omega_3 = y^n(2) + ny^{n-1}(0) + \ldots$
$\omega_3 = 2y^n$

Putting back into the degenerate eqn
$\omega_1 - \omega_2 - \omega_3 = 0$
$y^{n+2} - (y^{n+1}(2x+1) + 2ny^n) - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$
$y^{n+2} = y^{n+1}(2x+1) - 2ny^n + 2y^n$
$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$
$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1) \Rightarrow$ Proven

Scanned by CamScanner

Question 2

a) $y = x^3 e^{4x}$

Using Leibnitz theorem

$u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u''' = 64e^{4x}$ $u^{iv} = 256e^{4x}$

$v = x^3$ $v' = 3x^2$ $v'' = 6x$ $v''' = 6$ $v^{iv} = 0$

Hence $u^n = 4^n e^{4x}$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^2$

$+ \frac{n(n-1)(n-2)(n-3)}{4} u^{n-4} v^2 + \ldots$

$y^n = 4^n e^{4x}(x^3) + n4^{n-1}e^{4x}(3x^2) + \frac{n(n-1)}{2!} 4^{n-3} e^{4x}(6x) + \frac{n(n-1)(n-2)}{3!}$

$4^{n-3} e^{4x}(6) + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4}(0)^2$

$y^n = 4^n x^3 e^{4x} + n3x^2 4^{n-1} e^{4x} + 3xn(n-1)4^{n-2} + n(n-1)(n-2)4^{n-3}e^{4x}$ to

$y^n = e^{4x}4^{n-3}(x^3 4^3 + 3nx^2 4^2 + 3xn(n-1)4 + n(n-1)(n-2))$

$y^n = e^{4x}4^{n-3}(64x^3 + 48x^2 n + 12x(n-1) + n(n-1)(n-2))$

$y^5 = e^{4x}4^{5-3}(64x^3 + 48x^2(5) + 12x(5)(5-1) + 5(5-1)(5-2))$

$y^5 = e^{4x}4^2(64x^3 + 240x^2 + 240x + 36)$

$y^5 = 16e^{4x}(64x^3 + 240x^2 + 240x + 36)$

2b) $x^2 \frac{d^2y}{dx^2} + n\frac{dy}{dx} + y = 0$

$x^2 y^x + xy' + y = 0$

$w_1 + w_2 + w_3 = 0$

$W_1 = x^2 y''$

$u = y''$ $u' = y'''$ $u'' = y^{iv}$ $u''' = y^v$ Hence $u^n = y^{n+2}$

$v = x^2$ $v' = 2x$ $v'' = 2$ $v''' = 0$

$w_1 = u^n(v) + nu^{n-1}(v') + \frac{n(n-1)}{2!} u^{n-2}(v'') + \frac{n(n-1)(n-2)}{3!} u^{n-3} v''' + \ldots$

$w_1 = y^{n+2}(x^2) + ny^{n+1}(2x) + \frac{n(n-1)}{2 \times 1} y^n(2) + 0$

$w_1 = x^2 y^{n+2} + 2xny^{n+1} + y^n n(n-1)$

$W_2 = xy'$

$u = y'$ $\quad u' = y''$ $\quad u'' = y'''$ Hence $u^n = y^{n+1}$

$v = x$ $\quad v' = 1$ $\quad v'' = 0$

$W_2 = u^n(v) + nu^{n-1}(v') + \frac{n(n-1)}{2!}u^{n-2}v^2$

$W_2 = y^{n+1}(x) + ny^n(1) + 0$

$W_2 = xy^{n+1} + ny^n$

$W_3 = y$

$u = y$ $\quad u' = y'$ $\quad$ Hence $u^n = y^n$

$v = 1$ $\quad v' = 0$

$W_3 = u^n(v) + nu^{n-1}(v')$

$= y(1) + 0 = y$

$W_1 + W_2 + W_3 = 0$

$y^{n+2}(x^2) + ny^{n+1}(2x) + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n(n-1) + n + 1)$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2+1) = 0$