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[illegible]/1027
Chem Eng

1a $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2x+1 \frac{d}{dx}(e^{x^2+x}) + e^{x^2+x} \frac{d(2x+1)}{dx}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

1b $y'' - y'(2x+1) - 2y = 0$

Using leibniz theorem

$w_1 = y''$

$w_2 = y'(2x+1)$

$w_3 = 2y$

degenerate eqn: $w_1 - w_2 - w_3 = 0$

w_1

If $U = y''$ $\quad U' = y'''$ hence $U^n = y^{n+2}$

$V = 1$ $\quad V' = 0$

$w_1 = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V'' + \ldots$

$w_1 = y^{n+2}(1) + ny^{n+1}(0)$

$w_1 = y^{n+2}$ --- (1)

w_2

$w_2 = y'(2x+1)$

$U = y'$ $\quad U' = y''$ $\quad U'' = y'''$ hence $U^n = y^{(n+1)}$

$V = 2x+1$ $\quad V' = 2$ $\quad V'' = 0$

$w_2 = U^nV + nU^{n-1}V' + n(n-1)/2!\, U^{n-2}V'' + \frac{n(n-1)(n-2)U^{n-3}V'''}{3!}$

$+ \ldots$

$w_2 = y^{n+1}(2x+1) + ny^n(2) + \frac{n(n-1)}{2!}y^{n-1}(0)$

$w_2 = y^{n+1}(2x+1) + 2ny^n + 0$

$w_2 = y^{n+1}(2x+1) + 2ny^n$

w_3

$u = y \quad u' = y' \quad$ Hence $\quad u^n = y^n$

$v = 2 \quad v' = 0$

$w_3 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \ldots$

$w_3 = y^n(2) + n y^{n-1}(0) + \ldots$

$w_3 = 2y^n$

$w_1 - w_2 - w_3 = 0$

$y^{n+2} - \left(y^{n+1}(2x+2) + 2ny^n\right) - 2y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

Question 2

2a $\quad y = x^3 e^{4x}$

using Leibniz theorem

$u = e^{4x} \quad u' = 4e^{4x} \quad u'' = 16e^{4x} \quad u''' = 64e^{4x} \quad u^{iv} = 256e^{4x}$

$v = x^3 \quad v' = 3x^2 \quad v'' = 6x \quad v''' = 6$

Hence $\quad u^n = 4^n e^{4x}$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v'''$

$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^{iv} + \ldots$

$y^n = 4^n e^{4x}(x^3) + n4^{n-1}e^{4x}(3x^2) + \frac{n(n-1)}{2!} 4^{n-2} e^{4x}(6x)$

$+ \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x}(6) + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4}(0)$

$y^n = 4^n x^3 e^{4x} + 12x^2 4^{n-1} e^{4x} + 6x n(n-1) 4^{n-2} + n(n-1)(n-2) 4^{n-3} e^{4x}$
$+ 0$

$y^n = e^{4x} 4^{n-3} (64x^3 + 48x^2 n + 12x n(n-1) + n(n-1)(n-2))$

$y^5 = e^{4x} 4^{5-3} (64x^3 + 48x^2(5) + 12x(5)(5-1) + 5(5-1)(5-2))$

$y^5 = e^{4x} 4^2 (64x^3 + 240x^2 + 240x + 36)$

$y^5 = 16e^{4x} (64x^3 + 240x^2 + 240x + 36)$

3) $\frac{d^2y}{dx^2} + \frac{x\,dy}{dx} + y$

$x^2 y'' + xy' + y = 0$

$w_1 + w_2 + w_3 = 0$

$w_1 = x^2 y^{n+1}$

$u = y''$ $\quad u' = y'''$ $\quad u'' = y^{iv}$ $\quad u''' = y^{v}$

$v = x^2$ $\quad v' = 2x$ $\quad v'' = 2$ $\quad v''' = 0$

$w_1 = u^n(v) + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v''' + \ldots$

$w_1 = y^{n+2}(x^2) + n y^{n+1}(2x) + \frac{n(n-1)}{2 \times 1} y^n (2) + 0$

$w_1 = x^2 y^{n+2} + 2xn y^{n+1} + y^n n(n-1)$

$W_2 = xy'$

$u = y'$ $\quad u' = y''$ $\quad u'' = y'''$ $\quad$ Hence $u^n = y^{n+1}$

$v = x$ $\quad v' = 1$ $\quad v'' = 0$

$w_2 = u^n(v) + n u^{n-1}(v') + \frac{n(n-1)}{2!} u^{n-2} v''$

$w_2 = y^{n+1}(x) + n y^n (1) + 0$

$w_2 = x y^{n+1} + n y^n$

$w_3 = y$

$\cancel{u = y}$ $\cancel{v = x}$ $\cancel{u' = y''}$ $\cancel{v' = 1}$ $\cancel{v'' = y'''}$ $\cancel{v'' = 0}$ ~~Hence~~ $\cancel{u^n = y^{n+1}}$

$w_3 = y$

$u = y \qquad u' = y' \qquad$ Hence $u^n = y^n$

$v = 1 \qquad v' = 0$

$w_3 = u^n(v) + n u^{n-1}(v')$

$= y(1) + 0 = y$

$w_1 + w_2 + w_3 = 0$

$y^{n+2}(x^2) + n y^{n+1}(2x) + n(n-1)y^n + x y^{n+1} + n y^n$

$+ y^n = 0$

$x^2 y^{n+2} + x y^{n+1}(2n+1) + y^n(n(n-1) + n + 1)$

$x^2 y^{n+2} + x y^{n+1}(2n+1) + y^n(n^2+1) = 0$