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17/ENG01/014
Chemical Engineering
ENG 381 (LMS 2)

Question 1

1(a) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2x+1 \frac{d}{dx}(e^{x^2+x}) + e^{x^2+x} \frac{d}{dx}(2x+1)$ (product Rule)

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y \Rightarrow$ proves

1(b) $y'' - y'(2x+1) - 2y = 0$

Using Leibnitz Theorem

$w_1 = y''$

$w_2 = y'(2x+1)$

$w_3 = 2y$

degenerate eqn $= w_1 - w_2 - w_3 = 0$

w_1

$u = y''$ $\quad u' = y'''$ $\quad$ hence $u^n = y^{n+2}$

$v = 1$ $\quad v' = 0$

$w_1 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \ldots$

$w_1 = y^{n+2}(1) + n y^{n+1}(0)$

$w_1 = y^{n+2}$ ——— (i)

w_2

$w_2 = y'(2x+1)$

$u = y'$ $\quad u' = y''$ $\quad u''' = y'''$, hence $u^n = y^{n+1}$

$v = 2x+1 \quad v' = 2 \quad v'' = 0$

$$w_2 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \dots$$

$$w_2 = y^{n+1}(2x+1) + n y^n (2) + \frac{n(n-1)}{2!} y^{n-2}(0) + \dots$$

$$w_2 = y^{n+1}(2x+1)\ 2n y^n + 0$$

$$w_2 = y^{n+1}(2x+1) + 2n y^n \quad - \text{ii)}$$

$w_3)$

$u = y \qquad u' = y' \qquad$ Hence $u^n = y^n$

$v = 2 \qquad v' = 0$

$$w_3 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \dots$$

$$w_3 = y^n(2) + n y^{n-1}(0) + \dots$$

$$w_3 = 2y^n \quad - - - \text{(iii)}$$

Putting back into degenerate eqn.

$$w_1 - w_2 - w_3 = 0$$

$$y^{n+2} - \left(y^{n+1}(2x+1) + 2n y^n\right) - 2y^n = 0$$

$$y^{n+2} - y^{n+1}(2x+1) - 2n y^n - 2y^n = 0$$

$$y^{n+2} = y^{n+1}(2x+1) + 2n y^n + 2y^n$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1) \Rightarrow \text{proven.}$$

Question 2

2a) $y = x^3 e^{4x}$

Using Leibnitz Theorem

$U = e^{4x}$, $U_1 = 4e^{4x}$, $U_{11} = 16e^{4x}$ $U''' = 64e^{4x}$ $U^{iv} = 256e^{4x}$

$V = x^3$, $V' = 3x^2$ $V'' = 6x$ $V''' = 6$ $V^{iv} = 0$

Hence $U^n = 4^n e^{4x}$

$$y^n = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^2$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!}U^{n-4}V^2 + \dots$$

$$y^n = 4^n e^{4x}(x^3) + n4^{n-1}e^{4x}(3x^2) + \frac{n(n-1)}{2!}4^{n-3}e^{4x}(6x) + \frac{n(n-1)(n-2)}{3!}4^{n-3}e^{4x}(6)$$
$$+ \frac{n(n-1)(n-2)(n-3)}{4!}4^{n-4}(0)^2$$

$$y^{(n)} = 4^n x^3 e^{4x} + n3x^2 4^{n-1} e^{4x} + 3xn(n-1)4^{n-2} + n(n-1)(n-2)4^{n-3}e^{4x} + 0.$$

$$y^n = e^{4x}4^{n-3}(x^3 4^3 + 3nx^2 4^2 + 3xn(n-1)4 + n(n-1)(n-2))$$

$$y^n = e^{4x}4^{n-3}(64x^3 + 48x^2 n + 12xn(n-1) + n(n-1)(n-2))$$

$$y^5 = e^{4x}4^{5-3}(64x^3 + 48x^2(5) + 12x(5)(5-1) + 5(5-1)(5-2))$$

$$y^5 = e^{4x}4^2(64x^3 + 240x^2 + 240x + 36)$$

$$y^5 = 16e^{4x}(64x^3 + 240x^2 + 240x + 36)$$

2b) $x^2\frac{d^2y}{dx^2} + \frac{n\,dy}{dx} + y = 0$

$x^2y^x + xy' + y = 0$

$w_1 + w_2 + w_3 = 0$

$w_1 = x^2y'''$

$U = y''$ $U' = y'''$ $U'' = y^{iv}$ $U''' = y^v$ Hence, $U^n = y^{n+2}$

$V = x^2$ $V' = 2x$ $V'' = 2$ $V''' = 0$

$$w_1 = U^n(V) + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}(V'') + \frac{n(n-1)(n-2)}{3!}U^{n-3}V''' + \dots$$

$$w_1 = y^{n+2}(x^2) + ny^{n+1}(2x) + \frac{n(n-1)}{2\times1}y^n(2) + 0$$

$$w_1 = x^2y^{n+2} + 2xny^{n+1} + y^n n(n-1)$$

$w_2 = xy'$

$u = y'$ $\quad u' = y''$ $\quad u'' = y'''$ $\quad$ Hence $u^n = y^{n+1}$

$v = x$ $\quad v' = 1$ $\quad v'' = 0$

$$w_2 = u^n(v) + nu^{n-1}(v') + \frac{n(n-1)}{2!} u^{n-2} v^2$$

$$w_2 = y^{n+1}(x) + ny^n(1) + 0$$

$$w_2 = xy^{n+1} + ny^n$$

$w_3 = y$

$u = y$ $\quad u' = y'$ $\quad$ Hence $u^n = y^n$

$v = 1$ $\quad v' = 0$

$$w_3 = u^n(v) + nu^{n-1}(v')$$

$$= y(1) + 0 = y$$

$$w_1 + w_2 + w_3 = 0$$

$$y^{(n+2)}(x^2) + ny^{n+1}(2x) + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$$

$$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n(n-1) + n + 1)$$

$$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2+1) = 0$$

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