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17/ENG02/012
Computer Engineering
ENG 381
Maths Assignment 2

(1) Question 1

a $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2x+1 \frac{d}{dx}(e^{x^2+x}) + e^{x^2+x}\frac{d}{dx}(2x+1)$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y.$

b $y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem

$W_1 = y''$

$W_2 = y'(2x+1)$

$W_3 = 2y$

degenerate eqn $= W_1 - W_2 - W_3 = 0$

W_1

$U = y''$ $U' = y'''$ hence $U^n = y^{n+2}$

$V = 1$ $V' = 0$

$W_1 = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \ldots$

$W_1 = y^{n+2}(1) + ny^{n+1}(0)$

$W_1 = y^{n+2}$ — (1)

W_2

$W_2 = y'(2x+1)$

$U = y'$ $U' = y''$ $U''' = y'''$, Hence

$V = 2x+1$ $V' = 2$ $V'' = 0$ $U^n = y^{n+1}$

$W_2 = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^3 + \ldots$

$W_2 = y^{n+1}(2x+1) + ny^n(2) + \frac{n(n-1)}{2!}y^{n-2}(0) + \ldots$

$$W_2 = y^{n+1}(2x+1) + 2ny^n + 0$$

$$W_2 = y^{n+1}(2x+1) + 2ny^n \quad \text{--- (ii)}$$

W_3

$u = y \qquad u' = y' \qquad$ Hence $u^n = y^n$

$v = 2 \qquad v' = 0$

$$W_3 = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \ldots$$

$$W_3 = y^n(2) + ny^{n-1}(0) + \ldots$$

$$W_3 = 2y^n \quad \text{——— (iii)}$$

Putting base into the degenerate eqn.

$$W_1 - W_2 - W_3 = 0$$

$$y^{n+2} - (y^{n+1}(2x+1) + 2nyn) - 2yn = 0$$

$$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$$

$$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$$

Question 2

2a $y = x^3 e^{4x}$

Using Leibnitz theorem

$U = e^{4x}$, $U' = 4e^{4x}$, $U'' = 16e^{4x}$, $U''' = 64e^{4x}$, $U^{iv} = 256e^{4x}$

$V = x^3$, $V' = 3x^2$, $V'' = 6x$, $V''' = 6$, $V^{iv} = 0$

Hence $U^n = 4^n e^{4x}$

$$y^n = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^2$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!}U^{n-4}V^2 + \dots$$

$$y^n = 4^n e^{4x}(x^3) + n4^{n-1}e^{4x}(3x^2) + \frac{n(n-1)}{2!}4^{n-3}e^{4x}(6x)$$

$$+ \frac{n(n-1)(n-2)}{3!}4^{n-3}e^{4x}(6) + \frac{n(n-1)(n-2)(n-3)}{4!}4^{n-4}(0)^2$$

$$y^n = 4^n x^3 e^{4x} + n3x^2 4^{n-1}e^{4x} + 3xn(n-1)4^{n-2} + n(n-1)(n-2)4^{n-3}e^{4x} + 0.$$

$$y^n = e^{4x}4^{n-3}(x^3 4^3 + 3nx^2 4^2 + 3xn(n-1)4 + n(n-1)(n-2))$$

$$y^n = e^{4x}4^{n-3}(64x^3 + 48x^2 n + 12xn(n-1) + n(n-1)(n-2))$$

$$y^5 = e^{4x}4^{5-3}(64x^3 + 48x^2(5) + 12x(5)(5-1) + 5(5-1)(5-2))$$

$$y^5 = e^{4x}4^2(64x^3 + 240x^2 + 240x + 36)$$

$$y^5 = 16e^{4x}(64x^3 + 240x^2 + 240x + 36)$$

2b $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$W_1 + W_2 + W_3 = 0$

$W_1 = x^2y''$

$U = y''$, $U' = y'''$, $U'' = y^{iv}$, $U''' = y^v$ Hence

$V = x^2$, $V' = 2x$, $V'' = 2$, $V''' = 0$, $U^n = y^{n+2}$

$$W_1 = U^n(V) + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}(V'') +$$

$$\frac{n(n-1)(n-2)}{3!}U^{n-3}V''' + \dots$$

$W_1 = y^{n+2}(x^2) + ny^{n+1}(2x) + \frac{n(n-1)}{2x} y^n (2) + 0$

$W_1 = x^n y^{n+2} + 2xny^{n+1} + y^n n(n-1)$

$W_2 = xy'$

$U = y' \quad U' = y'' \quad U'' = y'''$ Hence $U^n = y^{n+1}$

$V = x \quad V' = 1 \quad V'' = 0$

$W_2 = U^n(V) + nU^{n-1}(V') + \frac{n(n-1)}{2!} U^{n-2} V^2$

$W_2 = y^{n+1}(x) + ny^n(1) + 0$

$W_2 = xy^{n+1} + ny^n$

$W_3 = y$

$U = y \quad U' = y'$ Hence

$V = 1 \quad V' = 0 \quad U^n = y^n$

$W_3 = U^n(V) + nU^{n-1}(V')$

$= y(1) + 0 = y$

$W_1 + W_2 + W_3 = 0$

$y^{n+2}(x^2) + ny^{n+1}(2x) + n(n-1)y^n + xy^{n+1} + ny^n$

$+ y^n = 0$

$x^2 y^{n+2} + xy^{n+1}(2n+1) + y^n(n(n-1) + n + 1)$

$x^2 y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2+1) = 0$