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17/ENG02/028
COMPUTER ENGINEERING

1a. $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2x + \frac{d(e^{x^2+x})}{dx} + e^{x^2+x}\frac{d(2x+1)}{dx}$ (Product Rule)

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y \Rightarrow$ Proven

1b. $y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem

$w_1 = y''$

$w_2 = y'(2x+1)$

$w_3 = 2y$

Degenerate eqn $= w_1 - w_2 - w_3 = 0$

w_1

$u = y''$ $\quad u' = y'''$ hence $u^n = y^{n+2}$

$v = 1$ $\quad v' = 0$

$w_1 = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \ldots$

$w_1 = y^{n+2}(1) + ny^{n+1}(0)$

$w_1 = y^{n+2}$ ——— i

w_2

$w_2 = y'(2x+1)$

$u = y'$ $\quad u' = y''$ $\quad u''' = y'''$, Hence $u^n = y^{n+1}$

$v = 2x+1$ $\quad v' = 2$ $\quad v'' = 0$

$w_2 = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)}{3!}u^{n-3}v^3 + \ldots$

$w_2 = y^{n+1}(2x+1) + ny^n(2) + \frac{n(n-1)}{2!}y^{n-2}(0) + \ldots$

$W_2 = y^{n+1}(2x+1) + 2ny^n + 0$

$W_2 = y^{n+1}(2x+1) + 2ny^n$ ——— ii

<u>W_3</u>

$u = y$ $\quad u' = y'$ $\quad$ Hence $u^n = y^n$

$v = 2$ $\quad v' = 0$

$$W_3 = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \dots$$

$W_3 = y^n(2) + ny^{n-1}(0) + \dots$

$W_3 = 2y^n$ ——— iii

Putting back into the degenerated equation

$W_1 - W_2 - W_3 = 0$

$y^{n+2} - (y^{n+1}(2x+1) + 2ny^n) - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

2a.) $y = x^3 e^{4x}$

Using Leibnitz theorem

$u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$ $\quad u''' = 64e^{4x}$

$u^{iv} = 256e^{4x}$

$v = x^3$ $\quad v' = 3x^2$ $\quad v'' = 6x$ $\quad v''' = 6$ $\quad v^{iv} = 0$

Hence $u^n = 4^n e^{4x}$

$$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)}{3!}u^{n-3}v^2$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!}u^{n-4}v^2 + \dots$$

$$y^n = 4^n e^{4x}(x^3) + n4^{n-1}e^{4x}(3x^2) + \frac{n(n-1)}{2!}4^{n-3}e^{4x}(6x)$$

$$\cancel{+\frac{n(n-1)(n-2)(4^{n-3})}{4!}e^{4x}(6)} + n(n-1)4^{n-3}e^{4x}(6x)$$

$$+\frac{n(n-1)(n-2)4^{n-3}e^{4x}(6)}{3!}+\frac{n(n-1)(n-2)(n-3)4^{n-4}(0)}{4!}$$

$$y^n = 4^n x^3 e^{4x} + n3x^2 4^{n-1}e^{4x} + 3xn(n-1)4^{n-2} + n(n-1)(n-2)4^{n-3}e^{4x} + 0$$
$$y^n = e^{4x}4^{n-3}(x^3 4^3 + 3nx^2 4^2 + 3xn(n-1)4 + n(n-1)(n-2))$$
$$y^n = e^{4x}4^{n-3}(64x^3 + 48x^2 n + 12xn(n-1) + n(n-1)(n-2))$$
$$y^5 = e^{4x}4^{5-3}(64x^3 + 48x^2(5) + 12x(5)(5-1) + 5(5-1)(5-2))$$
$$y^5 = e^{4x}4^2(64x^3 + 240x^2 + 240x + 36)$$
$$y^5 = 16e^{4x}(64x^3 + 240x^2 + 240x + 36)$$

26.) $x^2\frac{d^2y}{dx^2} + n\frac{dy}{dx} + y = 0$

$$x^2y'' + xy' + y = 0$$
$$w_1 + w_2 + w_3 = 0$$

$w_1 = x^2y''$

$u = y''$ $u' = y'''$ $u'' = y^{iv}$ $u''' = y^{v}$ Hence, $u^n = y^{n+2}$

$v = x^2$ $v' = 2x$ $v'' = 2$ $v''' = 0$

$$w_1 = u^n(v) + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}(v'') + \frac{n(n-1)(n-2)}{3!}u^{n-3}v''' + \dots$$

$$w_1 = y^{n+2}(x^2) + ny^{n+1}(2x) + \frac{n(n-1)}{2\times1}y^n(2) + 0$$

$$w_1 = x^2y^{n+2} + 2xny^{n+1} + y^n n(n-1)$$

$w_2 = xy'$

$u = y'$ $u' = y''$ $u'' = y'''$ Hence $u^n = y^{n+1}$

$v = x$ $v' = 1$ $v'' = 0$

$$w_2 = u^n(v) + nu^{n-1}(v') + \frac{n(n-1)}{2!}u^{n-2}v^2$$

$$w_2 = y^{n+1}(x) + ny^n(1) + 0$$
$$w_2 = xy^{n+1} + ny^n$$

$\underline{W_3 = y}$

$u = y \quad u' = y'$ Hence $u^n = y^n$

$v = 1 \quad v' = 0$

$W_3 = u^n(v) + n u^{n-1}(v')$

$= y(1) + 0 = y$

$W_1 + W_2 + W_3 = 0$

$y^{n+2}(x^2) + n y^{n+1}(2x) + n(n-1)y^n + x y^{n+1} + n y^n + y^n = 0$

$x^2 y^{n+2} + x y^{n+1}(2n+1) + y^n(n(n-1) + n + 1) = 0$

$x^2 y^{n+2} + x y^{n+1}(2n+1) + y^n(n^2+1) = 0$