$y^{n+2}(x^2) + ny^{n+1}(2x) + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n(n-1) + n + 1)$

$x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2+1) = 0$

2. Using Leibnitz Theorem, given that:

i) $y = x^3e^{4x}$ determine $y^{(5)}$.

$u = e^{4x}$ $\qquad v = x^3$

$u^n = 4^n e^{4x}$ $\qquad v' = 3x^2$

$u^{n-1} = 4^{(n-1)}e^{4x}$ $\qquad v'' = 6x$

$u^{n-2} = 4^{(n-2)}.e^{4x}$ $\qquad v''' = 6.$ $\qquad u^{(n-3)} = 4^{(n-3)}.e^{4x}$

Hence $u^n = 4^n e^{4x}$

$y^n = u^n v + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}v^{(2)}}{2!} + \frac{n(n-1)(n-2)u^{(n-3)}v^{(3)}}{3!} + \frac{n(n-1)(n-2)(n-3)u^{(n-4)}}{4!}$

$v^{(4)} + \ldots$

$y^n = 4^n e^{4x}.x^3 + n[4^{(n-1)}e^{4x}.3x^2] + \frac{n(n-1)}{2!}6x.4^{(n-2)}e^{4x} + \frac{n(n-1)(n-2)}{3!}$

$4^{(n-3)}.e^{4x} + 0$

$y^{(n)} = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)}e^{4x} + 3nx(n-1)4^{(n-2)}e^{4x} + n(n-1)(n-2)$

$4^{(n-3)}e^{4x} + 0$

$y^n = y^5$ i.e $n = 5$

$y^5 = x^3 4^{(5)}e^{4x} + 3(5)x^2 4^{(5-1)}e^{4x} + 3(5)(5-1)x.4^{(5-2)}.e^{4x} + 5(5-1)$

$(5-2)4^{(5-3)}e^{4x}$

$y^{(5)} = 1024x^3 + 3840x^2e^{4x} + 3840x^{4x} + 960e^{4x}$

ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$

Show that

$x^2y^{(n+2)} + (2n+1)y^{(n+1)} + (n^2+1)y^n = 0.$

ASSIGNMENT 2

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17/eng01/030
Chemical Engineering.
Eng 281 Assignment 2 Solution.

1. If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$ and prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

$\cancel{y' = (2x+1)e^{x^2+x}}$

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y' = 2x+1 \frac{\partial}{\partial x}(e^{x^2+x}) + e^{x^2+x}\frac{\partial}{\partial x}(2x+1)$

$y''' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y = e^{x^2+x}$ $\therefore y'' = y'(2x+1) + 2y \longrightarrow$ Proven.

ii) $y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem

$w_1 = y''$, $w_2 = y'(2x+1)$ $w_3 = 2y$.

For w_1: $u = y^{(2)}$ $v = 1$

$u^n = y^{(n+2)}$ $v' = 0$

For w_2: $u = y'$ $v = 2x+1$

$u^n = y^{(n+1)}$ $v' = 2$

$u^{(n-1)} = y^n$ $v'' = 0$.

For w_3: $u = y$ $v = 2$

$u^n = y^n$ $v' = 0$.

Using Leibnitz theorem:

$y^{(n)} = u^n v + n u^{(n-1)} v + \frac{n(n-1)}{2} u^{(n-2)} v^{(2)} + \frac{n(n-1)(n-2)}{3} u^{(n-3)} v^{(3)} + \dots$

$y^{(n)} = y^{(n+2)} \cdot 1 \longrightarrow y^{(n+2)}\ (w_1)$

$y^{(n)} = y^{(n+1)}(2x+1) + n(y^n) \times 2 \Rightarrow \cancel{2x+1}\ y^{n+1}(2x+1) + 2yn \rightarrow$ For w_2

$y^{(n)} = y^n \times 2 = 2y^n \rightarrow$ For w_3

Solution.

$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y^{(0)} = 0$ Let x^2y'' be ω_1

$\omega_1 = x^2y''$ $v = x^2$

$u = y''$ $v' = 2x$

$u^n = y^{(n+2)}$ $v'' = 2$

$u^{(n-1)} = y^{(n+1)}$ $v''' = 0$

$u^{(n-2)} = y^n$ $v^{(4)} = 0$

Let xy' be ω_2

$u = y'$ $v = x$

$u^n = y^{(n+1)}$ $v' = 1$

$u^{(n-1)} = y^n$ $v'' = 0$

Let y be ω_3

$u = y$ $v = 1$

$u^n = y^n$ $v' = 0$

$y^n = u^nv + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}v''}{2!} + \frac{n(n-1)(n-2)u^{(n-3)}v'''}{3!} + \dots$

For ω_1: $y^{(n+2)} \cdot x^2 + n(y^{(n+1)} \cdot 2x) + \frac{n(n-1)}{2!} y^n \cdot 2 + 0$

For ω_2: $y^{(n+1)} \cdot x^2 + n(y^{(n+1)} \cdot 2x) + 0$

For ω_3: y^n

$y^{(n+2)}x^2 + 2nxy^{(n+1)} + \frac{n(n-1)}{2} \cdot 2y^n + y^{(n+1)} \cdot x + nyn + y^n = 0$

$x^2y^{(n+2)} + 2nxy^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + nyn + y^n = 0$

$\cancel{x^2y^n}$ $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$