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17/ENG01/021

CHEMICAL ENGINEERING

Question 1

If $Y = e^{x^2+x}$

Sol

$Y = e^{x^2+x}$

Differentiating

$Y = e^u$

let $u = x^2+x$

$du/dx = 2x+1$

$dY/du = e^u$

Differentiating Using Function of function

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^u(2x+1) = (2x+1)e^{x^2+x}$

But $Y' = \frac{dy}{dx}$

$Y' = (2x+1)e^{x^2+x}$

Differentiating further.

$\frac{d^2y}{dx^2} = Y'' = \frac{d}{dx}(2x+1)e^{x^2+x}$

let $U = 2x+1 \Rightarrow du/dx = 2$

$V = e^{x^2+x}$

Differentiating using function of function

$dv/dx = dv/dz \times dz/dx$

let $z = x^2+x$

$dz/dx = 2x+1$, $dv/dz = e^z = e^{x^2+x}$

$\frac{dv}{dx} = e^z \times (2x+1) = e^{(x^2+x)}(2x+1)$

$\frac{dv}{dx} = (2x+1)e^{x^2+x}$.

Differentiating Using Product Rule

$\frac{d^2y}{dx^2} = U\, dv/dx + V\, du/dx$

$$\frac{d^2y}{dx^2} = (2x+1)\left[(2x+1)e^{x^2+x}\right] + e^{(x^2+x)} \times 2$$

$$\frac{d^2y}{dx^2} = (2x+1)\left[(2x+1)e^{x^2+x}\right] + 2e^{x^2+x}$$

But $Y = e^{x^2+x}$ & $Y' = (2x+1)e^{x^2+x}$

$$\frac{d^2y}{dx^2} = Y'(2x+1) + 2Y$$

Since $\frac{d^2Y}{dx^2} = Y''$

$Y'' = Y'(2x+1) + 2Y$

$Y'(2x+1) + 2Y - Y'' = 0$

let $k_1 = Y'(2x+1)$

$k_2 = 2Y$

$k_3 = Y''$

For k_1

$U = Y'$, $V = 2x+1$

$U^{(n)} = Y^{(n+1)}$ $V' = 2$

$U^{(n-1)} = Y^{(n)}$ $V'' = 0$

For k_2

$U = Y$ $V = 2$

$U^n = Y^n$ $V' = 0$

For k_3

$U = Y''$ $V = 1$

$U^n = Y^{(n+2)}$ $V' = 0$

$U^{(n-1)} = Y^{(n+1)}$

Recall.

$$Y^n = U^nV + nU^{(n-1)}V' + \frac{n(n-1)}{2!}U^{(n-2)}V^2 + \frac{n(n-1)(n-2)}{3!}U^{(n-3)}V^3 + \cdots$$

$\Rightarrow Y^{(n+1)} \cdot (2x+1) + nY^n \times 2 + Y^n \times 2 - Y^{(n+2)} \times 1 + nY^{(n+1)} \times 0 = 0$

$\Rightarrow 2x+1 \cdot Y^{(n+1)} + 2nY^n + 2Y^n - Y^{(n+2)} = 0$

$Y^{(n+2)} = (2x+1) \cdot Y^{(n+1)} + 2nY^n + 2Y^n$

$Y^{(n+2)} = (2x+1) \cdot Y^{(n+1)} + 2(n+1)Y^n$

Question 2.

$y = x^3 e^{4x}$

$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^4$

$+ \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} u^{(n-5)} v^5 + \dots$

y^5

$u = e^{4x}$ $\quad$ $v = x^3$

$u^n = 4^n e^{4x}$ $\quad$ $v^{(1)} = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad$ $v^{(2)} = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad$ $v^{(3)} = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad$ $v^{(4)} = 0$

$y^n = 4^n e^{4x} \cdot x^3 + n 4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{(n-2)} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} \cdot 6$

$y^{(5)} = 4^{(5)} e^{4x} \cdot x^3 + 5[4^{(5-1)} e^{4x} \cdot 3x^2] + \frac{5(5-1)}{2!} 4^{(5-2)} e^{4x} \cdot 6x + \frac{5(5-1)(5-2)}{3!} 4^{(5-3)} e^{4x} \cdot 6$

$y^{(5)} = 4^{(5)} x^3 e^{4x} + 5[4^{(4)} \cdot 3x^2 e^{4x}] + \frac{5(4)}{2!} 4^3 \cdot 6x e^{4x} + \frac{5(4)(3)}{3!} 4^{(2)} e^{4x} \cdot 6$

$y^{(5)} = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + \frac{7680}{2!} x e^{4x} + \frac{5760}{3!} e^{4x}$

$y^5 = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

$y^5 = e^{4x} [1024x^3 + 3840x^2 + 3840x + 960]$.

Question 3.

$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$.

$x^2 y'' + x y' + y = 0$

let

$k_1 = x^2 y''$

$k_2 = x y'$

$k_3 = y$.

$W_1 = x^2 y''$

$u = y^2 \qquad V = x^2$

$u^n = y^{(n+2)} \qquad V' = 2x$

$u^{(n-1)} = y^{(n+1)} \qquad V^2 = 2$

$u^{(n-2)} = y^n \qquad V^3 = 0$

$W_2 = x y'$

$u = y' \qquad V = x$

$u^n = y^{(n+1)} \qquad V' = 1$

$u^{(n-1)} = y^n \qquad V^2 = 0$

$W_3 = y$

$u = y \qquad V = 1$

$u^n = y^n \qquad V' = 0$

$\Rightarrow$ ~~#~~ From Leibnitz theorem.

$$u^{(n)}V + n u^{(n-1)}V' + \frac{n(n-1)}{2!} u^{(n-2)} V^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} V^{(3)} + \cdots$$

$$\Rightarrow y^{(n+2)} \cdot x^2 + n(y^{(n+1)} \cdot 2x) + \frac{n(n-1)}{2!} y^n \cdot 2 + \cancel{\frac{n(n-1)(n-2)}{3!} y^n}$$

$$\Rightarrow y^{(n+2)} \cdot x^2 + n y^{(n+1)} 2x + \frac{n(n-1)}{2!} y^n \cdot 2 + y^{(n+1)} \cdot x + n y^n \cdot 1 + y^n \cdot 1 = 0$$

$$x^2 y^{(n+2)} + 2xn y^{(n+1)} + \frac{n(n-1)}{2!} 2 y^n + x y^{(n+1)} + n y^n + y^n = 0$$

$$x^2 y^{(n+2)} + 2xn y^{(n+1)} + n(n-1) y^n + x y^{(n+1)} + n y^{(n)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + 2xn y^{(n+1)} + x y^{(n+1)} + n(n-1) y^n + n y^n + y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + [n(n-1) + n + 1] y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$$