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Mechanical Engineering ENG 381

1) If $y = e^{x^2+x}$
$y'' = y'(2x+1) + 2y$ and hence prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

$y = e^{x^2+x}$
$y' = 2x+1 \cdot (e^{x^2+x})$
$y'' = 2(e^{x^2+x}) + (2x+1)(2x+1)\, e^{x^2+x}$
$y'' = (2x+1)(2x+1)e^{x^2+x}) + 2(e^{x^2+x})$
$y'' = y'(2x+1) + 2(y)$

$A = y''$ $A_n = y^{n+2}$
$B = y'(2x+1)$ $B_n = y^{(n+1)}(2x+1) + ny^n(2)$
$C = 2y$ $C_n = 2y^n$
$\therefore y^{n+2} = y^{(n+1)}(2x+1) + 2ny^n + 2y^n$
$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

2) Using lebinz's theorem
i) $y = x^3 e^{4x}$ determine y^5
$u = x^3$ $u' = 3x^2$ $u'' = 6x$ $u''' = 6$
$u = e^{4x}$ $u' = 4e^{4x}$ $u'' = 16e^{4x}$ $u''' = 4^n e^{4x}$
$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \ldots$

$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} \cdot 6x +$

$\frac{n(n-1)(n-2)}{3!} \cdot 4^{n-3} e^{4x} 6 + 0$

$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 n 4^{n-1} e^{4x} + 3n(n-4)4^{n-2} e^{4x} x +$
$n(n-1)(n-2) \cdot 4^{n-3} e^{4x}$.
$y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2(5)4^{5-1} e^{4x} + 3(5)(5-1)4^{(5-2)} e^{4x} +$
$5(5-1)(5-2) \cdot 4^{(5-3)} e^{4x}$
$y^5 = 1024 e^{4x} \cdot x^3 + 3840 x^2 e^{4x} + 3840 e^{4x} \cdot x + 960 e^{4x}$

$y^5 = 64e^{4x}(16x^3+60x^2+60x+15)$,,

b) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx^2} + y = 0$ show that $x^2y^{n+2} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Solution

$= x^2y'' + xy' + y = 0$

$A = x^2y''$, $A^n = y^{n+2}x^2 + ny^{n+1}\cdot 2x + \frac{n(n-1)}{2!}\cancel{(n-2)}y^n \cdot 2 + 0$

$A^n = y^{n+2} + ny^{n+1}\cdot 2x + n(n-1)(n-2)y^n$

$B = xy'$ $B^n = y^{n+1}x + ny^n$

$C = y$ $C^n = y^n$

$\therefore$

$= y^{n+2}x^2 + ny^{n+1}2x + (n^2-n)y^n + y^{n+1}x + ny^n + y^n$

$x^2(y^{n+2}) + xy^{n+1}(2n+1) + y^n(n^2-n+n+1)$

$= x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$.

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