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Mechatronic Engineering

ENG 381 (LMS2)

(1a)

$$y = e^{x^2+x}$$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = (2x+1)\frac{\delta}{\delta x}(e^{x^2+x}) + e^{x^2+x}\frac{\delta}{\delta x}(2x+1)$$ (Product Rule)

$$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$$

$$y' = (2x+1)e^{x^2+x}$$

$$y = e^{x^2+x}$$

$$y'' = y'(2x+1) + 2y$$ => Proven

(1b)

$$y'' - y'(2x+1) - 2y = 0$$

Using leibnitz theorem

$$w_1 = y''$$

$$w_2 = y'(2x+1)$$

$$w_3 = 2y$$

Degenerate eqn $= w_1 - w_2 - w_3 = 0$

w_1

$U = y''$ $\quad U' = y'''$ $\quad$ hence $U^n = y^{n+2}$

$V = 1$ $\quad V' = 0$

$$w_1 = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \ldots\ldots$$

$$w_1 = y^{n+2}(1) + ny^{n+1}(0)$$
$$w_1 = y^{n+2}$$

<u>W_2</u>

$$W_2 = y'(2x+1)$$

$u = y'$, $u' = y''$, $u''' = y'''$ Hence $u^n = y^{n+1}$
$v = 2x+1$, $v' = 2$, $v'' = 0$

$$W_2 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2$$
$$+ \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \dots$$

$$W_2 = y^{n+1}(2x+1) + ny^n(2) + \frac{n(n-1)}{2!} y^{n-2}(0) + \dots$$

$$W_2 = y^{n+1}(2x+1) + 2n y^n + 0$$

$$W_2 = y^{n+1}(2x+1) + 2n y^n$$

<u>W_3</u>

$u = y$, $u' = y'$, Hence $u^n = y^n$
$v = 2$, $v' = 0$

$$W_3 = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \dots$$
$$W_3 = y^n(2) + ny^{n-1}(0) + \dots$$
$$W_3 = 2y^n \quad \dots \quad (iii)$$

Putting back into the degenerate equation

$$W_1 - W_2 - W_3 = 0$$

$$y^{n+2} - \left(y^{n+1}(2x+1) + 2n y^n\right) - 2y^n = 0$$

$$y^{n+2} - y^{n+1}(2x+1) - 2n y^n - 2y^n = 0$$

$$y^{n+2} = y^{n+1}(2x+1) + 2n y^n + 2y^n$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1) \Rightarrow \text{Proven}$$

Question 2.

2a) $y = x^3 e^{4x}$

Using Leibnitz theorem

$U = e^{4x}, \; U' = 4e^{4x}, \; U'' = 16e^{4x}, \; U''' = 64e^{4x}, \; U'''' = 256e^{4x}$

$V = x^3, \; V' = 3x^2, \; V'' = 6x, \; V''' = 6, \; V'''' = 0$

Hence $U^n = 4^n e^{4x}$

$$y^n = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!}U^{n-3}V^3$$
$$+ \frac{n(n-1)(n-2)(n-3)}{4!}U^{n-4}V^2 + \ldots\ldots$$

$$y^n = 4^n e^{4x}(x^3) + n4^{n-1}e^{4x}(3x^2) + \frac{n(n-1)}{2!}4^{n-2}e^{4x}(6x)$$
$$+ \frac{n(n-1)(n-2)}{3!}4^{n-3}e^{4x}(6) + \frac{n(n-1)(n-2)(n-3)}{4!}4^{n-4}(0)^2$$

$$y^n = 4^n x^3 e^{4x} + n3x^2 4^{n-1}e^{4x} + 3xn(n-1)4^{n-2} + n(n-1)(n-2)4^{n-3} \cdot e^{4x} + 0$$

$$y^n = e^{4x}4^{n-3}(x^3 4^3 + 3n x^2 4^2 + 3xn(n-1)4 + n(n-1)(n-2))$$
$$y^n = e^{4x}4^{n-3}(64x^3 + 48x^2 n + 12x\,n(n-1) + n(n-1)(n-2))$$
$$y^n = e^{4x}4^{5-3}(64x^3 + 48x^2(5) + 12x(5)(5-1) + 5(5-1)(5-2))$$
$$y^5 = e^{4x}4^2(64x^3 + 240x^2 + 240x + 36)$$
$$y^5 = 16e^{4x}(64x^3 + 240x^2 + 240x + 36)$$

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2b) $x^2 \frac{d^2y}{dx^2} + n\frac{dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$w_1 + w_2 + w_3 = 0$

$w_1 = x^2 y'''$

$u = y''$, $u' = y'''$, $u'' = y^{iv}$, $u''' = y^{v}$ Hence $u^n = y^{n+2}$

$v = x^2$, $v' = 2x$, $v'' = 2$ $v''' = 0$

$w_1 = u^n(v) + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} (v^{iv}) + \frac{n(n-1)(n-2)}{3!} u^{n-3}$

$v''' \ldots\ldots$

$w_1 = y^{n+2}(x^2) + n y^{n+1}(2x) + \frac{n(n-1)}{2} y^n (2) + 0$

$w_1 = x^2 y^{n+2} + 2xn y^{n+1} + y^n n(n-1)$

$w_2 = xy'$

$u = y'$, $u' = y''$, $u'' = y'''$ Hence $u^n = y^{n+1}$

$v = x$, $v' = 1$ $v'' = 0$

$w_2 = u^n(v) + n u^{n-1}(v') + \frac{n(n-1)}{2!} u^{n-2} v^2$

$w_2 = y^{n+1}(x) + n y^n (1) + 0$

$w_2 = x y^{n+1} + n y^n$

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$$w_3 = u^n (v) + n u^{n-1} (v')$$
$$= y(1) + 0 = y$$

$$w_1 + w_2 + w_3 = 0$$
$$y^{n+2} (x^2) + n y^{n+1} (2x) + n(n-1) y^n + x y^{n+1} + n y^n + y^n = 0$$

$$x^2 y^{n+2} + x y^{n+1} (2n+1) + y^n (n(n-1) + n+1)$$

$$x^2 y^{n+2} + x y^{n+1} (2n+1) + y^n (n^2+1) = 0.$$