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17/ENG02/029
Computer Engineering
ENG381

Assignment 2

1 $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x} . (2x+1) + 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= 2e^{x^2} + (2x+1)^2 e^{x^2+x}$

but $y'' = 2e^{x^2} + (2x+1)^2 e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$A = y''$, $A' = y'''$ $A^n = y^{2+?}$

$B = y'(2x+1)$

$u = y'$, $y^n = y^{n+1}$

$v = 2x+1$

$v' = 2$

$v'' = 0$

$B^n = (y^{n+1})(2x+) + n(y^n)(2) + 0$

$B^? = (2x+1)y^{n+1} + 2ny^?$

$C = 2y$

$C^n = 2y^?$

$A^n = B^n + C^?$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^?$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

2i $y = x^3 e^{4x}$, determine $y^{(5)}$

Let $u = e^{4x}$ $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u^n = 4^n e^{4x}$

let $v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v^{iv} = 0$

by Leibnitz theorem

$y^n = 4^n e^{4x} x x^3 + n \times 4^{n-1} e^{4x} \times 3x^2 + \frac{n(n-1)}{2!} \times 4^{n-2} e^{4x} \times 6x +$

$\frac{n(n-1)(n-2)x}{3!} \times 4^{n-3} e^{4x} \times 6 + c.$

$y^n = 4^n e^{4x} \times x^3 + 3x^2 n \times 4^{n-1} e^{4x} + 3n(n-1) \times 4^{n-2} e^{4x} \times x + n(n-1)(n-2) \times 4^{n-3} e^{4x}$

$y^5 = 4^5 e^{4x} \times x^3 + 3x^2(5) \times 4^{5-1} e^{4x} + 3(5)(4) \times 4^3 e^{4x} \times x + 5(4)(3) \times 4^2 e^{4x}$

$y^5 = 1024 e^{4x} \times x^3 + 3840 e^{4x} \times x^2 + 3840 e^{4x} \times x + 960 e^{4x}$

$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$

Q: $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$A = x^2 y''$

$u = y'' ,\ u^n = y^{n+2}$

$v = x^2,\ v' = 2x,\ v'' = 2,\ v''' = 0$

$A^n = (y^{n+2})x^2 + n(y^{n+1})2x + \frac{n(n-1)}{2!}(y^n) \times 2 + 0$

$A^n = x^2 y^{(n+2)} + 2xny^{(n+1)} \dots 2! \dots + n(n-1)y^n$

$B = xy'$

$u = y' \quad u^n = y^{n+1}$

$v = x \quad v' = 1 \quad v'' = 0$

$B^n = (y^{n+1})x + n(y^n) \times 1 + 0$

$= xy^{(n+1)} + ny^n$

$c = y$

$c^n = y^?$

$A^n + B^n + C^? = 0$

$= x^2 y^{(n+2)} + 2xny^{(n+1)} + (n^2-n)y^? + xy^{(n+1)} + ny^n + y^? = 0$

$= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$= x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$