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17/SCI01/084

ENG 281 Assignment II

1. If $y = e^{x^2+x}$

Show that

$y'' = y'(2x+1) + 2y$ and hence, prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$

2. Using the Leibnitz theorem, given that

(i) $y = x^3 e^{2x}$, determine $y^{(n)}$

(ii) $x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$, show that

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$

Solution

1) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$y'' = (2x+1)y' + \cancel{2e^{x^2+x}}\ 2y$

$y'' = y'(2x+1) + 2y$

letting $w_1 = y'(2x+1)$

$w_2 = 2y$

$w_1 \Rightarrow$ $u = y'$ $\quad v = 2x+1$

$u' = y''$ $\quad v' = 2$

$u^n = y^{(n+1)}$ $\quad v'' = 0$

$w_2 \Rightarrow$ $u = y$ $\quad v = 2$

$u^n = y^n$ $\quad v' = 0$

For $w_1 \Rightarrow$

$= y^{(n+1)}(2x+1) + ny^{(n+1-1)}.2 + \frac{n(n-1)}{1\times2}y^{(n+1-2)}$

$= y^{(n+1)}(2x+1) + ny^{(n)}(2)$

ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$\omega_1 = x^2y''$

$u = y''$ $\qquad v = x^2$

$u^n = y^{(n+2)}$ $\qquad v' = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\qquad v'' = 2$

$u^{(n-2)} = y^n$ $\qquad v''' = 0$

$\omega_2 = xy$

$u = y'$ $\qquad v = x$

$u^n = y^{(n+1)}$ $\qquad v' = 1$

$u^{(n-1)} = y^n$ $\qquad v'' = 0$

$\omega_3 = y^n \Rightarrow y^{(n)}$

$\omega_1 = y^{(n+2)}.x^2 + n.y^{(n+1)}.2x + \frac{n(n-1)}{2!}y^n.2$

$= x^2y^{(n+2)} + 2nxy^{(n+1)} + n(n-1)y^n$

$\omega_2 = y^{(n+1)}.x + n.y^{(n)}.1$

$= xy^{(n+1)} + ny^{(n)}$

$\omega_3 = y^n$

$\therefore x^2y^{(n+2)} + 2nxy^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$

$\Rightarrow x^2y^{(n+2)} + \cancel{xy^{(n+1)}(2n+1)} + 2nxy^{(n+1)} + xy^{(n+1)} + n(n-1)y^{(n)} + ny^{(n)} + y^n$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + [n(n-1) + n+1]y^n = 0$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + (n^2 - n + n + 1)y^n = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

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$w_2 = y^n 2 + n y^{(n-1)} \cdot 0$

$+ y^{(n)} \cdot 2$

$w_1 + w_2 =$

$= y^{(n+1)}(2x+1) + n y^{(n)} 2 + y^n \cdot 2$

$= y^{(n+1)}(2x+1) + 2n y^{(n)} + 2y^{(n)}$

$= y^{(n+1)}(2x+1) + 2y^n(n+1)$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^n(n+1)$

$= y^{(n+1)}(2x+1) + 2(n+1)y^n$

$\therefore y^{(n+2)} = y^{(n+1)}(2x+1) + 2(n+1)y^n$

(2) : $y = x^3 e^{4x}$

$u = e^{4x}$ $\quad$ $v = x^3$

$u' = 4e^{4x}$ $\quad$ $v' = 3x^2$

$u'' = 16e^{4x}$ $\quad$ $v'' = 6x$

$u^n = 4^n e^{4x}$ $\quad$ $v''' = 6$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad$ $v'''' = 0$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$

$= 4^n e^{4x} \cdot x^3 + n \cdot 4^{(n-1)} e^{4x} \cdot 3x^2 + n(n-1) 4^{(n-2)} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{1 \times 2 \times 3}$

$4^{(n-3)} e^{4x} \cdot 6 + \cdots$

$= 4^n x^3 e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + 3n(n-1)x 4^{(n-2)} e^{4x} + n(n-1)(n-2) 4^{(n-3)} e^{4x}$

Find $y^{(n)} = y^5$

$y^5 = 4^5 x^3 e^{4x} + 3(5)x^2 4^{(5-1)} e^{4x} + 3(5)(5-1)x 4^{(5-2)} e^{4x} + 5(5-1)(5-2) 4^{(5-3)} e^{4x}$

$= 4^5 x^3 e^{4x} + 15x^2 4^4 e^{4x} + 60x 4^3 e^{4x} + 60 \cdot 4^2 e^{4x}$

$= 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

$\therefore y^5 = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

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