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CHEMICAL ENGINEERING
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ENG 381

ASSIGNMENT II

1.) $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$ and hence prove that:

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$$

Solution

$y = e^{x^2+x}$

Differentiate $y = y'$ & y''

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x} \cdot 2$$

$$y'' = (2x+1)y' + y \cdot 2$$

$$\therefore y'' = y'(2x+1) + 2y$$

Finding the nth derivative of y''

i.e $y^{(n+2)} = y^{(n+1)}(2x+1) + ny^{(n)} \cdot 2 + 2y^{(n)}$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$$

$$\therefore y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^{(n)}(n+1)$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$$

2i) $y = x^3 e^{4x}$, determine $y^{(5)}$

Solution

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^{4} + \frac{n(n-1)(n-2)}{3!} \cdot u^{(n-3)} v''' + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^{(4)} + \dots$$

taking

$u = e^{4x}$ $\quad v = x^3$

$u^n = 4^n e^{4x}$ $\quad v' = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad v'' = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad v''' = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad v^{(4)} = 0$

$u^{(n-4)} = 4^{(n-4)} e^{4x}$ $\quad v^{(5)} = 0$

$u^{(n-5)} = 4^{(n-5)} e^{4x}$

Substituting values into general solution

$$y^n = 4^n e^{4x} \cdot x^3 + n\left[4^{(n-1)} e^{4x} \cdot 3x^2\right] + \frac{n(n-1)}{2!} \cdot 4^{(n-2)} e^{4x} \cdot 6x + \dots + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} \cdot 6 + \frac{n(n-1)(n-2)(n-3)}{4!} \cdot 4^{(n-4)} e^{4x} \cdot 0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + \frac{n(n-1)}{2!} 6x \cdot 4^{(n-2)} e^{4x}$$

$$+\frac{n(n-1)(n-2)}{3\times 2\times 1}\cdot 6\cdot 4^{(n-3)}e^{4x}+0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + \frac{n(n-1)}{\cancel{2}!}\overset{3}{\cancel{6}}x\cdot 4^{(n-2)}e^{4x} + \frac{6n(n-1)(n-2)}{6}\cdot 4^{(n-3)}e^{4x}+0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + 3nx(n-1)4^{(n-2)}e^{4x} + n(n-1)(n-2)4^{(n-3)}e^{4x}+0$$

$y^n = y^5$, i.e $n=5$, sub value $n=5$ into the eqn above.

$$y^5 = x^3 4^{(5)} e^{4x} + 3(5)x^2 4^{(5-1)} e^{4x}\, 3(5)(5-1)x\cdot 4^{(5-2)}e^{4x} + 5(5-1)(5-2)4^{(5-3)}e^{4x}+0$$

$$y^5 = 1024x^3e^{4x} + 15(256)x^2e^{4x} + 15(4)(64)xe^{4x} + 5(4)(3)(16)e^{4x}+0$$

$$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}+0$$

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2(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that

$$x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1) y^{(n)} = 0$$

Solution

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$$x^2 y'' + xy' + y = 0$$

Taking $a_1 = x^2 y''$, $a_2 = xy'$, $a_3 = y$

For a_1

$u = y''$ $\quad v = x^2$

$y^{(n)} = y^{(n+2)}$ $\quad v' = 2x$

$y^{(n-1)} = y^{(n+1)}$ $\quad v'' = 2$

$y^{(n-2)} = y^{(n)}$ $\quad v''' = 0$

a_2

$u = y'$ $\quad v = x$

$y^{(n)} = y^{(n+1)}$ $\quad v' = 1$

$y^{(n-1)} = y^{n}$ $\quad v'' = 0$

$\underline{a_3}$

$u = y \qquad v = 1$

$u^n = y^n \qquad v' = 0$

$y^{(n)} = u^{(n)}v + n u^{(n+1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' +$

$\frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \frac{n(n-1)(n-2)(n-3)}{4!}$

$u^{(n-4)} v^{(4)} + \ldots$

$\therefore a_1^{(n)} = y^{(n+2)} \cdot x^2 + n [y^{(n+1)} \cdot 2x] + \frac{n(n-1)}{2!}$

$\cdot y^n \cdot 2 + 0$

$a_2^{(n)} = y^{(n+1)} \cdot x + n(y^n \cdot 1) + 0$

$a_3^{(n)} = y^n \cdot 1 + 0$

Substitute values of $a_1^{(n)}$, $a_2^{(n)}$, $a_3^{(n)}$ in the general equation

$x^2 y'' + xy' + y = 0$

$y^{(n+2)} \cdot x^2 + 2nx\, y^{(n+1)} + \frac{n(n-1)}{2 \times 1} \cdot 2 y^n + y^{(n+1)} \cdot x$

$+ ny^n + y^n = 0$

$$x^2 y^{(n+2)} + 2nx\, y^{(n+1)} + \frac{n(n-1)\cdot \cancel{2}}{\cancel{2}} y^n + x\, y^{(n+1)} + ny^n$$

$$+ y^n = 0$$

$$x^2 y^{(n+2)} + 2nx\, y^{(n+1)} + n(n-1) y^n + xy^{(n+1)} + ny^n$$

$$+ y^n = 0$$

$$x^2 y^{(n+2)} + 2nx^{(n+1)} + xy^{(n+1)} + n(n-1) y^n + ny^n + y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1)x\, y^{(n+1)} + (n^2 - \cancel{n} + \cancel{n} + 1) y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1)x\, y^{(n+1)} + (n^2+1) y^n = 0$$