2. $y'(2x+1)+2y$

$= (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

from the above equation,

Part A,

$A = y''$, $A' = y'''$, $A^n = y^{2+n}$

Part B

$B = y'(2x+1)$

$u = y'$, $u^n = y^{n+1}$

$v = 2x+1$

i.) $y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2 (5) \cdot 4^4 e^{4x} + 3(5)(4) \cdot 4^3 e^{4x} \cdot x + (5)(4)(3) \cdot 4^2 e^{4x}$

$y^5 = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} \cdot x + 960 e^{4x}$

$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$ //

ii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ , Show that $x^2 y^{(n+2)} + (2n+1) x \, y^{(n+1)} + (n^2+1) y^n = 0$

for Part A

$A = x^2 y''$

$u = y''$ , $u^n = y^{n+2}$

$v = x^2$ , $v' = 2x$ , $v'' = 2$ , $v''' = 0$

$A^n = (y^{n+2}) x^2 + n (y^{n+1}) \cdot 2x + \underline{n(n-1)} \cdot (y^n) \, 2 + 0$

$\therefore A^n + B^n + C^n = 0$

$= x^2 y^{(n+2)} + 2xny^{(n+1)} + (n^2 - n)y^n + xy^{(n+1)} + ny^n + y^n = 0$

$= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$= x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$