ASITA OBOMISO
17/ENG06/014
MECHATRONICS ENGINEERING

→ $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x} \cdot (2x+1) + 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2ex^{1+x}$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$A = y'$, $A' = y'''$, $A^n = y^{2+n}$

$B = y'(2x+1)$

$u = y'$, $u^n = y^{n+1}$

$v = 2x+1$

$v' = 2$, $v'' = 2$

$B^n = (y^{n+1})(2x+1) + n(y^n)(2) + 0$

$B^n = (2x+1)y^{n+1} + 2ny^n$

$C = 2y$

$C^n = 2y^n$

$A^n = B^n + C^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

→ i, $y = x^3e^{4x}$, determine $y^{(3)}$

Let $u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 6e^{4x}$, $u^n = 4^ne^{4x}$

Let $v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v^{iv} = 0$

by leibnitz theorem

$y^n = 4^ne^{4x} \times x^3 + n \times 4^{n-1}e^{4x} \times 3x^2 + \frac{n(n-1)}{2!} \times 4^{n-2}e^{4x} \times 6x + \frac{n(n-1)(n-2)}{3!} \times$

$4^{n-3}e^{4x} \times 6 + c$

$$y^n = 4^n e^{4x} \times x^3 + 3x^2 n \times 4^{n-1} e^{4x} + 3n(n-1) \times 4^{n-2} e^{4x} \times x + n(n-1)(n-2) \times 4^{n-3} e^{4x}$$

$$y^5 = 4^5 e^{4x} \times x^3 + 3x^2(5) \times 4^{5-1} e^{4x} + 3(5)(4) \times 4^3 e^{4x} \times x + 5(4)(3) \times 4^2 e^{4x}$$

$$y^5 = 1024 e^{4x} \times x^3 + 3840 e^{4x} \times x^2 + 3840 e^{4x} \times x + 960 e^{4x}$$

$$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$$

ii, $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ Show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

$A = x^2 y''$

$u = y''$, $u^n = y^{n+2}$

$V = x^2$, $V' = 2x$, $V'' = 2$, $V''' = 0$

$A^n = (y^{n+2}) x^2 + n(y^{n+1}) 2x + \frac{n(n-1)}{2!} \times (y^n) \times 2 + 0$

$\cancel{A^n = x^2 y^{(n+2)}}$

$A^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1) y^n$

$B = xy'$

$u = y'$ $u^n = y^{n+1}$

$V = x$ $V' = 1$ $V'' = 0$

$B^n = (y^{n+1}) x + n(y^n) \times 1 + 0$

$= xy^{(n+1)} + ny^n$

$C = y$

$C^n = y^n$

$A^n + B^n + C^n = 0$

$= x^2 y^{(n+2)} + 2xny^{(n+1)} + (n^2 - n) y^n + xy^{(n+1)} + ny^n + y^n = 0$

$= x^2 y^{(n+2)} + xy^{(n+1)} (2n+1) + y^n (n^2 + n + n + 1) = 0$

$= x^2 y^{(n+2)} + (2n+1) xy^{(n+1)} + (n^2+1) y^n = 0$