ASSIGNMENT 2
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CIVIL ENGINEERING
ENG 381

1) If Solution

$y = e^{x^2+x}$

$e^{x^2} \cdot x$

$u = e^{x^2} \quad v = e^x$

$du/dx = 2xe^{x^2} \quad dv/dx = e^x$

$y' = e^{x^2} \cdot e^x + e^x \cdot 2xe^{x^2}$

$y' = e^{x^2+x} + 2xe^{x^2+x} \cdot e^{x^2+x}(2x+1)$

$u = e^{x^2+x} \quad v = 2x+1$

$du/dx = e^{x^2+x}(2x+1) \quad dv/dx = 2$

$y'' = e^{x^2+x} \cdot 2e + (2x+1)(2x+1)e^{x^2+x}$

recall $y = e^{x^2+x}$ and $y' = e^{x^2+x}(2x+1)$

$y'' = 2y + y'(2x+1) = y'(2x+1) + 2y$

$y'' = y'(2x+1) + 2y$

$u = y^{(1)} \quad v = 2x+1$

$u^n = y^{n+1} \quad v' = 2$

$y^{n-1} = y^{n-1+1} = y^n \quad v'' = 0$

$y^{(n+2)} = y^{(n+1)}2x+1 + ny^n \cdot 2 + 2y^n$

$y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$

ii) $y = e^{x^2+x}$

Differentiate

$y = y'$ and $y = y''$

$y = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x} \cdot 2$

$y'' = (2x+1)y' + y \cdot 2$

Therefore

$y'' = y'(2x+1) + 2y$

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finding the nth derivative of y''

$y^{(n+2)} = y^{(n+1)}(2x+1) + ny^{n} \cdot 2 + 2y^{n}$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{n}(n+1)$

Therefore

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^{(n)}(n+1)$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$

2) $y = x^3 e^4$ determine $y^{(5)}$

solution

$y^n = u^n v + nu^{(n-1)}v' + \frac{n(n-1)}{2} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \frac{n(n-1)(n-2)(n-3)}{3!}$

$+ u^{(n-4)} v^{iv} + \cdots$

Taking

$u = e^{4x}$ $\quad V = x^3$

$u^n = 4^n e^{4x}$ $\quad V' = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad V'' = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad V''' = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad V^4 = 0$

$u^{(n-4)} = 4^{(n-4)} e^{4x}$ $\quad V^5 = 0$

$u^{(n-5)} = 4^{n-5} e^{4x}$

substitute values in general ~~equation~~ solution

$y^n = 4^n e^{4x} \cdot x^3 + n[4^{(n-1)} e^{4x} \cdot 3x^2] + \frac{n(n-1)}{2!} \cdot 4^{(n-2)} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x}$

$\cdot 6 + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{(n-4)} e^{4x} \cdot 0$

$y^n = x^6 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + \frac{n(n-1)}{2 \times 1} 6x \cdot 4^{(n-2)} e^{4x} + \frac{n(n-1)(n-2)}{3 \times 2 \times 1} \cdot 6 \cdot 4^{(n-3)} e^{4x} + 0$

$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} \frac{n(n-1)}{2} 6x \cdot 4^{(n-2)} e^{4x} + \frac{6n(n-1)(n-2)}{6} \cdot 4^{(n-3)} e^{4x} + 0$

$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + 3nx(n-1) 4^{(n-2)} e^{4x} + n(n-1)(n-2) 4^{(n-3)} e^{4x} + 0$

$y^n = y^5$, i.e $n = 5$

$y^5 = x^3 4^{(5)} e^{4x} + 3(5) x^2 4^{(5-1)} e^{4x} + 3(5)(5-1)x \cdot 4^{(5-2)} e^{4x} + 5(5-1)(5-2) \cdot 4^{(5-3)} e^{4x}$

$y^5 = 1024x^3 e^{4x} + 3340 x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}$

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a) Solution

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$x^2y'' + xy' + y = 0$

$a_1 \quad a_2 \quad a_3$

taking a_1

$u = y''$ $\quad v = x^2$

$u^2 = y(n+2)$ $\quad v' = 2x$

$u(n-1) = y(n+1)$ $\quad v'' = 2$

$u^n = y^n$ $\quad v''' = 0$

Taking a_2

$u = y'$ $\quad v = x$

$u^n = y^{(n+1)}$ $\quad v' = 1$

$u^{(n-1)} = y^n$ $\quad v'' = 0$

Taking a_3

$u = y$ $\quad v = 1$

$u^n =$ $\quad v' = 0$

$$y^{(n)} = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \frac{n(n-1)(n-2)(n-3)}{4!} + u^{(n-4)} v^4 +$$

Therefore

$$a_1^{(n)} = y^{(n+2)} \cdot x^2 + n\left[y^{(n+1)} \cdot 2x\right] + \frac{n(n-1)}{2!} \cdot y^n \cdot 2 + 0$$

$$a_2^{(n)} = y^{(n+1)} \cdot x + n\left[y^n \cdot 1\right] + 0$$

$$a_3^{(n)} \cdot y^n \cdot 1 + 0$$

Substitute values of $a_1^{(n)} \cdot a_2^{(n)}, a_3^{(n)}$ into general equation

$$y^{(n+2)} \cdot x^2 + 2nx\, y^{(n+1)} + \frac{n(n-1)}{2} \cdot 2y^n + xy^{(n+1)} + ny^n + y^n = 0$$

$$x^2y^{(n+2)} + 2nx\, y^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n + y^n = 0$$

$$x^2y^{(n+2)} + 2nxy^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + ny^n + y^n = 0$$

$$x^2y^{(n+2)} + (2n+1)x\, y^{(n+1)} + (n^2 - n + n + 1)y^n = 0$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$$