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17/mhs01/083
Civil engineering
Assignment 2

(6) $y = e^{x^2+x}$

$e^{x^2} \cdot x$

$u = e^{x^2}$ $v = e^x$

$du/dx = 2x e^{x^2}$ $\frac{dv}{dx} = e^x$

$y'' = e^{x^2} \cdot e^x + e^x \cdot 2x e^{x^2}$

$y' = e^{x^2+x} + 2x e^{x^2+x} \cdot e^{x^2+x}(-2x+1)$

$u = e^{x^2+x}$ $v = 2x+1$ $dv/dx = 2$

$du/dx = e^{x^2+x}(2x+1)$

$y'' = e^{x^2+x} \cdot 2 e + (2x+1)(2x+1) e^{x^2+x}$

$y = e^{x^2+x}$ and $y' = e^{x^2+x}(2x+1)$

$y'' = 2y + y'(2x+1) = y'(2x+1) + 2y$

$y'' = y'(2x+1) + 2y$

$u = y^{(1)}$ $v = 2x+1$

$u^n = y^{n+1}$ $v' = 2$

$u^{n-1} = y^{n-1+1} = y^n$ $v'' = 0$

$y^{(n+2)} = y^{(n+1)} 2x + 1 + n y^n \cdot 2 + 2y^n$

$y^{(n+2)} = (2x+1) y^{n+1} + 2(n+1) y^n$

$y = e^{x^2+x}$

differentiate.

$y = y'$ and $y = y''$

$y = (2x+1) e^{x^2+x}$

$y'' = (2x+1)(2x+1) e^{x^2+x} + e^{x^2+x} \cdot 2$

$y'' = (2x+1) y' + y \cdot 2$

Therefore:

$y'' = y'(2x+1) + 2y$

find the nth derivative of y''

$y^{(n+2)} = y^{(n+1)}(2x+1) + n y^n \cdot 2 + 2 y^n$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}$

Therefore

$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2y^{(n)}$

$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2(n+1)\,y^{(n)}$

2) $y = x^3 e^4$ determine $y^{(5)}$

Soln

$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \frac{n(n-1)(n-2)(n-3)}{4!}$

$+ u^{(n-4)} v^{iv} + \cdots$

Taking

$u = e^{4x}$ $\quad v = x^3$

$u^n = 4^n e^{4x}$ $\quad v' = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad v'' = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad v''' = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad v^{iv} = 0$

$u^{(n-4)} = 4^{(n-4)} e^{4x}$ $\quad v^5 = 0$

$u^{(n-5)} = 4^{(n-5)} e^{4x}$

Substitute values in general solution

$y^n = 4^n e^{4x} \cdot x^3 + n[4^{(n-1)} e^{4x} \cdot 3x^2] + \frac{n(n-1)}{2!} \cdot 4^{(n-2)} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} \cdot 6 +$

$\frac{n(n-1)(n-2)(n-3)}{4!} 4^{(n-4)} e^{4x} \cdot 0$

$y^n = x^3 4^n e^{4x} + 3nx^{(n-1)} e^{4x} + \frac{n(n-1)}{2+1} 6x \cdot 4^{(n-2)} e^{4x} + \frac{n(n-1)(n-2)}{3+2+1} \cdot 6\, 4^{(n-3)} e^{4x} + 0$

$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} \frac{n(n-1)}{2} 6x \cdot 4^{(n-2)} e^{4x} + \frac{n(n-1)(n-2)}{6} \cdot 4^{(n-3)} e^{4x} + 0$

$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + 3nx(n-1) 4^{(n-2)} e^{4x} + n(n-1)(n-2) 4^{(n-3)} e^{4x} + 0$

$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + 3nx(n-1) 4^{(n-2)} e^{4x} + n(n-1)(n-2) 4^{(n-3)} e^{4x} + 0$

$y^n = y^5$, i.e $n = 5$

$y^5 = x^3 4^{(5)} e^{4x} + 3(5) x^2 4^{(5-1)} e^{4x} + 3(5)(5-1) x \cdot 4^{(5-2)} e^{4x} + 5(5-1)(5-2) 4^{(5-3)} e^{4x}$

$y^5 = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

Sol:

$$\frac{x^2 d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$x^2y'' + xy' + y = 0$

$a_1 \quad a_2 \quad a_3$

taking a_1

$U = y'' \quad \& \quad V = x^2$

$U^2 = y(n+2) \qquad U' = 2x$

$U^{(n-1)} = y(n+1) \qquad V'' = 2$

$U^n = y^n \qquad V''' = 0$

Taking a_2

$U = y' \quad V = x$

$U^n = y(n+1) \quad V' = 1$

$U(n-1) = y^n \quad V'' = 0$

Taking a_3

$U = y \quad V = 1$

$U_n = \quad V' = 0$

$$y^{(n)} = U^n V + nU^{(n-1)}V' + \frac{n(n-1)}{2!}U^{(n-2)}V'' + \frac{n(n-1)(n-2)}{3!}U^{(n-3)}V''' + \frac{n(n-1)(n-2)(n-3)}{4!}U^{(n-4)} + \dots$$

Therefore

$a_1^{(n)} = y^{n+2} \cdot x^2 + n[y^{(n+1)} \cdot 2x] + \frac{n(n-1)}{2!} y^n \cdot 2 + 0$

$a_2^{(n)} = y(n+1) \cdot x + n[y^n \cdot 1] + 0$

$a_3^{(n)} \cdot y^n \cdot 1 + 0$

Substitute value of $a_1^{(n)}, a_2^{(n)}, a_3^{(n)}$ in to general equation.

$y^{n+2} \cdot x^2 + 2nxy^{(n+1)} + \frac{n(n-1)}{2} \cdot 2y^n + xy^{(n+1)} + ny^n + y^n = 0$

$x^2y^{n+2} + 2nxy^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n + y^n = 0$

$x^2y^{(n+2)} + 2nxy^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + ny^n + y^n = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2 - n + n + 1)y^n = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$