ARCH TOCHUKWU · C.
17/ENG04/012
ELECTRICAL AND ELECTRONICS ENGINEERING

i. $Y = e^{x^2+x}$

$Y' = (2x+1)e^{x^2+x}$

$Y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$Y''' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

ii. $Y'(2x+1) + 2y$

$= (2x+1)e^{x^2+x} \cdot (2x+1) + 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

but $Y'' = Y'(2x+1) + 2y$

$\cdot Y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

from the above equation

Part A

$A = Y''$, $A' = Y'''$, $A^2 = Y^2 + n$

Part B

$B = Y'(2x+1)$

$U = Y'$ $\quad U^n = Y^{n+1}$

$V = 2x+1$ $\quad V = 2$

$V''' = 0$

$B^n = (Y^{n+1})(2x+1) + n(Y^n)(2) + 0$

$B^n = (2x+1)Y^{n+1} + 2nY^n$

Part C

$C = 2y$

$C^n = 2y^n$

$A^n = B^n + C^n$

$Y^{n+2} = (2x+1)Y^{n+1} + 2ny^n + 2y^n$

$Y^{n+2} = (2x+1)Y^{n+1} + 2Y^n(n+1)$

$Y^{n+2} = (2x+1)Y^{n+1} + 2(n+1)Y^n$

a) $y = x^3 e^{4x}$, $y^{(n)}$

let $u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u^n = 4^n e^{4x}$
$v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v^{iv} = 0$

Using Liebnitz Theorem

$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} \cdot 4^{n-2} e^{4x} \cdot 6x$

$+ \frac{n(n-1)(n-2)}{3!} \cdot 4^{n-3} e^{4x} \cdot 6 \cdots$

$y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2(5) \cdot 4^4 e^{4x} + 3(5)(4) \cdot 4^3 e^{4x} \cdot x +$
$(5)(4)(3) \cdot 4^2 e^{4x}$
$y^5 = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} \cdot x + 960 e^{4x}$

(ii) $x^2 \frac{dy}{dx} + x \frac{dy}{dx} + y = 0$, Show that $x^2 (y^{n+2}) + (2n+1)x\, y^{(n+1)} + (n^2+1) y^n = 0$

1st for Part A
$A = x^2 y''$
$u = y''$, $u^n = y^{n+2}$
$v = x^2$, $v' = 2x$, $v'' = 2$

$A^n = (y^{n+2}) x^2 + n(y^{n+1}) \cdot 2x + \frac{n(n-1)}{2!} \cdot y^n \cdot 2 + 0$

$A^n = x^2 y^{n+2} + 2xny^{(n+1)} + n(n-1) y^n$

for Part B
$B = xy'$
$u = y'$, $u^n = y^{n+1}$

$v = x$, $v' = 1$, $v'' = 0$
$B^n = (y^{n+1}) \cdot x + n(y^n) \cdot 1 + 0$
$= x y^{(n+1)} + n y^n$

For Part c

$c = y$

$c^n = y^n$

$A^n + B^n + C^n = 0$

$x^2 y^{(n+2)} + 2xny^{(n+1)} + (n^2 - n)y^n + xy^{(n+1)} + ny^n + y^n = 0$

$= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y(n^2 - n + n + 1) = 0$

$= x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$