ADEPOJU MARY ABIMBOLA
17/ENG03/004
CIVIL ENGINEERING
ENG 381

1) $y = e^{x^2+x}$

Differenciate y - y' & y''

$$\frac{dy}{dx} = 2xe^{x^2} \cdot e^{x} + e^{x^2} \cdot e^{x}$$

$$y' = (2x+1)\, e^{x^2+x}$$

$$u = e^{x^2+x} \qquad v = 2x+1$$

$$\frac{du}{dx} = e^{x^2+x}(2x+1) \qquad \frac{dv}{dx} = 2$$

$$y'' = e^{x^2+x} \cdot 2e + (2x+1)(2x+1)\, e^{x^2+x}$$

$$y'' = 2y + y'(2x+1) = y'(2x+1) + 2y$$

Where $y = e^{x^2+x}$

$y' = e\ e^{x^2+x}(2x+1)$

$$y'' = (2x+1)\, y' + y \cdot 2$$

$\therefore$

$$y'' = y'(2x+1) + 2y$$

Find the nth derivative of y''

$y'' = y'(2x+1) + 2y$

$u = y^{(1)} \qquad v = 2x+1$

$u^{n} = y^{n+1} \qquad v' = 2$

$u^{n-1} = y^{n-1+1} = y^{n} \qquad v'' = 0$

$y^{(n+2)} = y^{(n+1)}\ 2x+1 + ny^{n} \cdot 2 + 2y^{n}$

$y^{(n+2)} = (2x+1)\, y^{n+1} + 2(n+1)\, y^{n}$

$y = e^{x^2+x}$

~~Determine~~ $y^{(5)}$

Solution

$y = y'$ and $y = y''$

$y = (2x + 2)\, e^{x^2+x}$

$y'' = (2x+1)(2x+1)\, e^{x^2+x} + e^{x^2+x} \cdot 2$

$y'' - (2x+1)\, y' + y \cdot 2$

$\therefore\ y'' = y'(2x+1) + 2y$

Find the nth derivative of y''

$y^{(n+2)} = y^{(n+1)}(2x+1) + n y^{n} \cdot 2 + 2y^{n}$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{n}(n+1)$

$\therefore$

$y^{(n+2)} = (2x+1)\, y^{(n+1)} + 2y^{(n)}(n+1)$

$y^{(n+2)} = (2x+1)\, y^{(n+1)} + 2(n+1)\, y^{n}$

$y = x^3 e^4$

Determine $y^{(5)}$

Solution

$$y^{n} = u^{n} v + n u^{(n-1)} v' + \frac{n(n-1)}{2} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v'''$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^{iv} + \ldots$$

$u = e^{4x}$ $\quad v = x^3$

$u^{n} = 4^{n} e^{4x}$ $\quad v' = 3x^2$

$u(n-1) = 4(n-1)\, e^{4x}$, $\quad v'' = 6x$

$u(n-2) = 4(n-2)\, e^{4x}$, $\quad v''' = 6$

$u(n-3) = 4(n-3)\, e^{4x}$, $\quad v^{iv} = 0$

$u(n-4) = 4(n-4)\, e^{4x}$ $\quad v^{v} = 0$

$u(n-4) = 4(n-5)\, e^{4x}$

$$y^n = 4^n e^{4x} \cdot x^3 + n(4^{(n-1)} e^{4x} \cdot 3x^2) + \frac{n(n-1)}{2!} +$$

$$4^{(n-2)} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} \cancel{4(n-3} + \cancel{4(n-3)e^{4x}}$$

$$4^{(n-3)} e^{4x} \cdot 6 + \frac{(n(n-1)(n-2)(n-3)}{4!} + 4^{(n-4)} e^{4x} \cdot 0$$

$$y^n = x^6 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + \frac{n(n-1)}{2\times 1} + 6x \cdot 4^{(n-2)} e^{4x} +$$

$$\frac{n(n-1)(n-2)}{3\times 2\times 1} + 6 \cdot 4^{(n-3)} e^{4x} + 0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} \frac{n(n+1)}{2} + 6x \cdot 4^{(n-2)} e^{4x} +$$

$$\frac{6n(n-1)(n-2)}{6} + 4^{(n-3)} e^{4x} + 0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + 3nx(n-1) + 4^{(n-2)} e^{4x} +$$
$$n(n-1)(n-2) + 4^{(n-3)} e^{4x} + 0$$

$y^n = y^5$

$\therefore n = 5$

$$y^5 = x^3 4^5 e^{4x} + 3(5)x^2 4(^{5-1)} e^{4x} + 3(5)(5-1)x +$$
$$4(^{5-2)} e^{4x} + 5(5-1)(5-2) + 4^{(5-3)} e^{4x}$$

$$y^5 = 1024x^3 e^{4x} + 3340x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}$$

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3) $\frac{x^2 d^2 y}{dx^2} + \frac{x\,dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$a_1 \qquad a_2 \qquad a_3$

For a_1

$u = y''$, $v = x^2$

$u = y^{(n+2)}$ $\qquad v' = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\qquad v'' = 2$

$u^n = y^n$ $\qquad v''' = 0$

For a_2

$u = y'$ $\qquad v = x$

$u^n = y^{(n+1)}$ $\qquad v' = 1$

$u^{(n-1)} = y^n$ $\qquad v'' = 0$

For a_3

$u = y$ $\qquad v = 1$

$u^n = y^n$ $\qquad v' = 0$

$y^{(n)} = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!}$

$u^{(n-3)} v''' + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^{iv} + \ldots +$

$\therefore$

$a_1{}^{(n)} = y^{(n+2)} \cdot x^2 + n(y^{(n+1)} \cdot 2x) + \frac{n(n-1)}{2!} \cdot y^n \cdot 2 + 0$

$a_2{}^{(n)} = y^{(n+1)} \cdot x + n(y^n \cdot 1) + 0$

$a_3{}^{(n)} = y^n \cdot 1 + 0$

Sub α_1^n, α_2^n & α_3^n in G.S

$y^{(n+2)} \cdot x^2 + 2nx\, y^{(n+1)} + \frac{n(n-1)}{2}\, 2y^n + xy^{(n+1)} +$

$ny^n + y^n = 0$

$x^2 y^{(n+2)} + 2nx\, y^{(n+1)} + n(n-1)\, y^n + xy^{(n+1)} + ny^n + y^n = 0$

$x^2 y^{(n+2)} + 2nxy^{(n+1)} + xy^{(n+1)} + n(n-1)\, y^n + ny^n + y^n = 0$

$x^2 y^{(n+2)} + (2n+1)x\, y^{(n+1)} + (n^2 - n + n + 1)y^n = 0$

$x^2 y^{(n+2)} + (2n+1)\, xy^{(n+1)} + (n^2+1)\, y^n = 0$