Bello Mahmud Mohammed
17/ENG06/018
Civil

ENG 381

$\therefore y^5 = \cancel{64e} \; 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$.

b) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ show that $x^2 y^{n+2} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$.

Solution

$= x^2y'' + xy' + y = 0$.

$\cancel{A^n = y^{n+2}x^2 +}$

$A = x^2y''$, $A^n = y^{n+2}x^2 + ny^{n+1}\cdot 2x + \frac{n(n-1)(n+2)y^n \cdot 2}{2!} + 0$

$A^n = y^{n+2}x^2 + ny^{n+1}\cdot 2x + n(n-1)(n-2)y^n$

$B = xy'$ $\quad B^n = y^{n+1}x + ny^n$

$C = y$ $\quad C_n = y^n$

$\therefore$

$= y^{n+2}x^2 + ny^{n+1}2x + (n^2-n)y^n + y^{n+1}x + ny^n + y^n$

$\cancel{2x}$ $\quad x^2(y^{n+2}) + xy^{n+1}(2n+1) + y^n(n^2-n+n+1)$

$= x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$.

i) If $y = e^{x^2+x}$
$y'' = y'(2x+1) + 2y$ and hence prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

$y = e^{x^2+x}$
$y' = 2x+1\,(e^{x^2+x})$
$y'' = 2(e^{x^2+x}) + (2x+1)(2x+1)e^{x^2+x}$
$y'' = (2x+1)(2x+1)e^{x^2+x}) + 2(e^{x^2+x})$
$y'' = y'(2x+1) + 2(y)$.

$A = y''$ $A_n = y^{n+2}$
$B = y'(2x+1)$ $B_n = y^{(n+1)}(2x+1) + ny^n(2)$
$C = 2y$ $C_n = 2y^n$
$\therefore$ $y^{n+2} = y^{(n+1)}(2x+1) + 2ny^n + 2y^n$
$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

2) Using Leibnitz theorem.
i) $y = x^3e^{4x}$ determine y^5
$V = x^3$ $V' = 3x^2$ $V'' = 6x$ $V''' = 6$ $V^n = \cancel{x}$
$U = e^{4x}$ $U' = 4e^{4x}$ $U'' = 16e^{4x}$ $U^n = 4^ne^{4x}$

$y^n = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V'' + \ldots$

$y^n = 4^ne^{4x}.x^3 + n.4^{n-1}e^{4x}.3x^2 + \frac{n(n-1)4^{n-2}}{2!}e^{4x}.6x +$

$\frac{n(n-1)(n-2)}{3!}.4^{n-3}e^{4x}6 + 0$

$y^n = 4^ne^{4x}.x^3 + 3x^2n4^{n-1}e^{4x} + 3n(n-1)4^{n-2}e^{4x}x +$
$n(n-1)(n-2).4^{n-3}e^{4x}$

$y^5 = 4^5e^{4x}.x^3 + 3x^2(5)4^{5-1}e^{4x} + 3(5)(5-1)4^{(5-2)}e^{4x}x +$
$5(5-1)(5-2).4^{(5-3)}e^{4x}$

$y^5 = 1024e^{4x}x^3 + 3840x^2e^{4x} + 3840e^{4x}.x + 960e^{4x}$.