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 (1) 18/EN604/077
 ELECTRICAL/ELECTRONICS ENGINEERING
 ASSIGNMENT 2

(1) if $y = e^{x^2+2x}$

Show that

$$y'' = y'(2x+1) + 2y$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

Solution

(a) $y = e^{x^2+2x}$ (i)

~~$y' = (2x+1)(2x+1)e^{x^2+2x} + 2 \cdot e^{x^2+2x}$~~

$y' = (2x+1)e^{x^2+2x}$ (ii)

$y'' = (2x+1)(2x+1)e^{x^2+2x} + 2 \cdot e^{x^2+2x}$

Considering equations (i) and (ii)

$y'' = (2x+1)y' + 2 \cdot y //$

(b) From the equation above it has been proven that;
 $y'' = y'(2x+1) + 2y$

Let; $y'' = w_1$

$y'(2x+1) = w_2$

$2y = w_3$

From $w_1 = y''$

$u = y'' \quad v = 1$

$u^n = y^{n+2} \quad v = 0$

From $w_2 = y'(2x+1)$

$u = y' \quad v = 2x+1$

$u^n = y^{n+1} \quad v' = 2$

$u^{n+1} = y^n \quad v'' = 0$

From $2y = 2y$

$$u^n = y^n \quad v = 2$$

$$u^{(n-1)} = y^n \quad v' = 0$$

$$\omega_1 + \omega_2 + \omega_3$$

Using the formula Leibnitz formula

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \dots$$

$$\omega_1 = y^{n+2} \cdot 1$$

$$2!$$

$$\begin{aligned} \omega_2 &= y^{(n+1)} (2x+1) + n y^n \cdot 2 \\ &= y^{(n+1)} (2x+1) + 2n y^n \end{aligned}$$

$$\omega_3 = y^n \cdot 2$$

$$\omega_1 = \omega_2 + \omega_3$$

$$y^{n+2} = y^{(n+1)} (2x+1) + 2n y^n + 2y^n$$

$$y^{n+2} = y^{(n+1)} (2x+1) + y^n (2n+2)$$

$$y^{n+2} = y^{n+1} (2x+1) + 2(n+1) y^n$$

Q.E.D.

Question 2

(1) $y = x^2 e^{4x}$ Determine $y^{(5)}$

$$u = e^{4x} \quad v = x^2$$

$$u' = 4e^{4x} \quad v' = 2x$$

$$u^{n-1} = 4^{(n-1)} e^{4x} \quad v'' = 2$$

$$u^{n-2} = 4^{(n-2)} e^{4x} \quad v''' = 0$$

$$u^{n-3} = 4^{(n-3)} e^{4x} \quad v^{(4)} = 0$$

Using Leibnitz formula

$$y^{(5)} = u^5 v + \frac{5 \cdot 4}{2!} u^{(4)} v' + \frac{5 \cdot 4 \cdot 3}{3!} u^{(3)} v'' + \dots$$

$$\frac{5 \cdot 4}{4!} u^{(4)} v''$$

$$4!$$

$$y^n \Rightarrow 4^n e^{4x} \cdot x^3 + n(4^{n-1} e^{4x}) \cdot 3x^2 + n(n-1)(4^{n-2} e^{4x}) \cdot 3x + \frac{n(n-1)(n-2)}{2!} (4^{n-2} e^{4x}) \cdot 6x + \frac{n(n-1)(n-2)}{3!} (4^{n-3} e^{4x}) \cdot 6 + 0$$

$$y'' = 4^n e^{4x} \cdot x^2 + n(4^{n-1} e^{4x}) \cdot 3x^2 + n(n-1)(4^{n-2} e^{4x}) \cdot 3x + n(n-1)(n-2)$$

$$y''' = 4^n e^{4x} \cdot x + 3x^2 \cdot n(4^{n-1} e^{4x}) + 3x \cdot n(n-1)(4^{n-2} e^{4x}) + n(n-1)(n-2)$$

$$y^{(5)} = x^2 4^5 e^{4x} + 3x \cdot 5(4^{5-1} e^{4x}) + 3x(5(5-1)) 4^{5-2} e^{4x} + 5(5-1)(5-2) \cdot (4^{5-3} e^{4x}) + 5(5-1)(5-2) \cdot 4x + 3x(5(5-1)) 4^{5-2} e^{4x} + 5(5-1)$$

$$y^5 = 1024x^5 e^{4x} + 3840x^3 e^{4x} + 3840x e^{4x} + 960e^{4x}$$

$$(c) x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$x^2 y'' + x y' + y = 0$$

$$\omega_1 = x^2 y''$$

$$\omega_2 = x y'$$

$$\omega_3 = y$$

$$\text{From } \omega_1 = x^2 y''$$

$$u = y'' ; v = x^2$$

$$u' = y''' ; v' = 2x$$

$$u^{n-1} = y^{n+1} ; v'' = 2$$

$$u^{n-2} = y^n ; v''' = 0$$

$$\text{From } \omega_2 = x y' ; u = y' ; v = x$$

$$u' = y'' ; v' = 1$$

$$\text{From } \omega_3 = y ; u = y ; v = 1$$

$$u' = y' ; v' = 0$$

Using Leibnitz theorem

$$= u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)(n-3)}{3!} v^{(3)} + \dots$$

$$w_1 = y^{(n)} = y^{(n+2)} x^2 + n y^{(n+1)} 2x + \frac{n(n-1)}{2!} y^{(n)} \cdot 2$$

$$w_2 = y^{(n+1)} x + n y^{(n)}$$

$$w_3 = y^{(n)}$$

$$w_1 + w_2 + w_3$$

$$\begin{aligned} &= y^{(n+2)} x^2 + n y^{(n+1)} 2x + n(n-1) y^{(n)} + y^{(n+1)} x + n y^{(n)} + y^{(n)} \\ &= x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + (n^2 + n + 1) y^{(n)} \\ &= x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + (n^2 + 1) y^{(n)} \end{aligned}$$