

ENG 381 ASSIGNMENT 2

QUESTION 1

1a) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2x+1 \frac{d}{dx}(e^{x^2+x}) + e^{x^2+x} \frac{d}{dx}(2x+1)$ (Product Rule)

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y \Rightarrow$ Proven

1b) $y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem

$W_1 = y''$

$W_2 = y'(2x+1)$

$W_3 = 2y$

degenerate eqn = $W_1 - W_2 - W_3 = 0$

W_1

$U = y'' \quad U' = y''' \quad \text{hence } U^n = y^{n+2}$

$V = 1 \quad V' = 0$

$W_1 = U^n V + nU^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V^2 + \dots$

$W_1 = y^{n+2}(1) + ny^{n+1}(0)$

$W_1 = y^{n+2} \dots (1)$

W_2

$W_2 = y'(2x+1)$

$U = y' \quad U' = y'' \quad U'' = y''' \quad \text{Hence } U^n = y^{n+1}$

$V = 2x+1 \quad V' = 2 \quad V'' = 0$

$W_2 = U^n V + nU^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V^2 + \frac{n(n-1)(n-2)}{3!} U^{n-3} V^3 + \dots$

$W_2 = y^{n+1}(2x+1) + ny^n(2) + \frac{n(n-1)}{2!} y^{n-2}(0) + \dots$

$$W_2 = y^{n+1}(2x+1) + 2ny^n + 0$$

$$W_2 = y^{n+1}(2x+1) + 2ny^n \dots \dots \dots (ii)$$

W_3

$$u = y \quad u' = y' \quad \text{Hence } u'' = y''$$

$$v = 2 \quad v' = 0$$

$$W_3 = u''v + nu''v' + \frac{n(n-1)}{2!} u''v'' + \dots$$

$$W_3 = y''(2) + ny''(0) + \dots$$

$$W_3 = 2y'' \dots \dots \dots (iii)$$

Putting back into the degenerate eqn

$$W_1 - W_2 - W_3 = 0$$

$$y^{n+2} - [y^{n+1}(2x+1) + 2ny^n] - 2y^n = 0$$

$$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$$

$$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1) \implies \text{Proven}$$

QUESTION 2

2a) $y = x^3 e^{4x}$

Using Leibnitz Theorem

$$U = e^{4x} \quad U' = 4e^{4x} \quad U'' = 16e^{4x} \quad U''' = 64e^{4x} \quad U^{(4)} = 256e^{4x}$$

$$V = x^3 \quad V' = 3x^2 \quad V'' = 6x \quad V''' = 6 \quad V^{(4)} = 0$$

Hence $U^n = 4^n e^{4x}$

$$y^n = U^n V + n U^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V'' + \frac{n(n-1)(n-2)}{3!} U^{n-3} V''' + \dots$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} U^{n-4} V^{(4)} + \dots$$

$$y^n = 4^n e^{4x} (x^3) + n 4^{n-1} e^{4x} (3x^2) + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} (6x) + \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} (6) + \dots$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4} (0)^2 + \dots$$

$$y^n = 4^n x^3 e^{4x} + n 3 x^2 4^{n-1} e^{4x} + 3 n(n-1) 4^{n-2} e^{4x} x + \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} (6) + 0$$

$$y^n = e^{4x} 4^{n-3} [x^3 4^3 + 3 n x^2 4^2 + 3 x n(n-1) 4 + n(n-1)(n-2)]$$

$$y^n = e^{4x} 4^{n-3} [64x^3 + 48x^2 n + 12x n(n-1) + n(n-1)(n-2)]$$

$$y^5 = e^{4x} 4^{5-3} [64x^3 + 48x^2(5) + 12x(5)(5-1) + 5(5-1)(5-2)]$$

$$y^5 = e^{4x} 4^2 [64x^3 + 240x^2 + 240x + 36]$$

$$y^5 = 16e^{4x} (64x^3 + 240x^2 + 240x + 36)$$

2b) $x^2 \frac{d^2 y}{dx^2} + n \frac{dy}{dx} + y = 0$

$$x^2 y'' + x y' + y = 0$$

$$W_1 + W_2 + W_3 = 0$$

$$W_1 = x^2 y''$$

$$U = y'' \quad U' = y''' \quad U'' = y^{(4)} \quad U''' = y^{(5)} \quad \text{Hence } U^n = y^{n+2}$$

$$V = x^2 \quad V' = 2x \quad V'' = 2 \quad V''' = 0$$

$$W_1 = U^n V + n U^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V'' + \frac{n(n-1)(n-2)}{3!} U^{n-3} V''' + \dots$$

$$W_1 = y^{n+2} (x^2) + n y^{n+1} (2x) + \frac{n(n-1)}{2!} y^n (2) + 0$$

$$W_1 = x^2 y^{n+2} + 2x n y^{n+1} + y^n n(n-1)$$

$$W_2 = xy'$$

$$u = y' \quad u' = y'' \quad u'' = y''' \quad \text{Hence } u^n = y^{n+1}$$

$$v = x \quad v' = 1 \quad v'' = 0$$

$$W_2 = u^n(v) + n u^{n-1}(v') + \frac{n(n-1)}{2!} u^{n-2} v''$$

$$W_2 = y^{n+1}(x) + n y^n(1) + 0$$

$$W_2 = x y^{n+1} + n y^n$$

$$W_3 = y$$

$$u = y \quad u' = y' \quad \text{Hence } u^n = y^n$$

$$v = 1 \quad v' = 0$$

$$W_3 = u^n(v) + n u^{n-1}(v')$$

$$W_3 = y^n(1) + 0 = y^n$$

$$W_1 + W_2 + W_3 = 0$$

$$y^{n+2}(x^2) + n y^{n+1}(2x) + n(n-1)y^n + x y^{n+1} + n y^n + y^n = 0$$

$$x^2 y^{n+2} + x y^{n+1}(2n+1) + y^n(n^2-1) + (n+1)y^n = 0$$

$$x^2 y^{n+2} + x y^{n+1}(2n+1) + y^n(n^2+1) = 0$$