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17/ENGG02/040

COMPUTER ENGR.

$$1) y = e^{x^2+x}$$

Show that  $y'' = y'(2x+1) + 2y$ , hence prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$$

soln

$$y = e^{x^2+x}$$

Differentiate  $y = y' + y''$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x} \cdot 2$$

$$y'' = (2x+1)y' + y \cdot 2$$

Therefore

$$y'' = y'(2x+1) + 2y$$

Finding the  $n^{\text{th}}$  derivative of  $y''$

$$\text{i.e. } y^{(n+2)} = y^{(n+1)}(2x+1) + n y^{(n)} \cdot 2 + 2y^{(n)}$$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$$

Therefore

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^{(n)}(n+1)$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$$

2.)

i)  $y = x^3 e^{4x}$ , determine  $y^{(5)}$   
 soln

$$y^n = U^n V + n U^{(n-1)} V' + \frac{n(n-1)}{2!} U^{(n-2)} V'' + \frac{n(n-1)(n-2)}{3!} U^{(n-3)} V''' + \frac{n(n-1)(n-2)}{3!} U^{(n-4)} V^{(4)} + \dots$$

$$\frac{n(n-1)(n-2)}{3!} U^{(n-4)} V^{(4)} + \dots$$

taking

$$\begin{aligned} U &= e^{4x} & V &= x^3 \\ U^n &= 4^n e^{4x} & V' &= 3x^2 \\ U^{(n-1)} &= 4^{(n-1)} e^{4x} & V'' &= 6x \\ U^{(n-2)} &= 4^{(n-2)} e^{4x} & V''' &= 6 \\ U^{(n-3)} &= 4^{(n-3)} e^{4x} & V^{(4)} &= 0 \\ U^{(n-4)} &= 4^{(n-4)} e^{4x} & V^{(5)} &= 0 \\ U^{(n-5)} &= 4^{(n-5)} e^{4x} & & \end{aligned}$$

Sub values in general soln

$$\begin{aligned} y^n &= 4^n e^{4x} \cdot x^3 + n [4^{(n-1)} e^{4x} \cdot 3x^2] + \frac{n(n-1)}{2!} \cdot 4^{(n-2)} e^{4x} \cdot 6x + \dots \\ &\dots + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} \cdot 6 + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{(n-4)} e^{4x} \cdot 0 \end{aligned}$$

$$y^n = x^3 4^n e^{4x} + \frac{3nx^2}{2 \times 1} 4^{(n-1)} e^{4x} + \frac{n(n-1)}{3 \times 2 \times 1} 6x \cdot 4^{(n-2)} e^{4x} + \frac{n(n-1)(n-2)}{3!} \cdot 6 \cdot 4^{(n-3)} e^{4x} + 0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + \frac{n(n-1)}{2!} 6x \cdot 4^{(n-2)} e^{4x} + \frac{6n(n-1)(n-2)}{6} 4^{(n-3)} e^{4x} + 0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + \frac{3nx}{2} (n-1) 4^{(n-2)} e^{4x} + n(n-1)(n-2) 4^{(n-3)} e^{4x} + 0$$

$y^5 = y^{(5)}$ , i.e.  $n=5$ , Sub value  $n=5$  into eqn above

$$y^5 = x^3 4^{(5)} e^{4x} + 3(5)x^2 4^{(5-1)} e^{4x} + \frac{3(5)(5-1)}{2} 4^{(5-2)} e^{4x} + 5(5-1)(5-2) 4^{(5-3)} e^{4x} + 0$$

$$y^5 = 1024$$

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$$x^2 \frac{d^2}{dx^2}$$

Show that  $x^2 y^{(n)}$

$$x^2 \frac{d^2 y}{dx^2}$$

$$x^2 y'' +$$

Considering:

$$U = y''$$

$$U^{(n)} = y^{(n+2)}$$

$$U^{(n-1)} = y^{(n+1)}$$

$$U^{(n-2)} = y^{(n)}$$

$$y^{(n)} = U^{(n)}$$

... + n

Therefore,

$$d_1^{(n)} =$$

$$d_2^{(n)} = y$$

$$d_3^{(n)} =$$

Subst

$$f^6 = 1024x^3e^{4x} + 15(256)x^2e^{4x} + 15(4)(64)x e^{4x} + 15(4)(3)(16)e^{4x} + 0$$

$$f^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x} + 0$$

(2ii)

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

Show that

$$x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^{(n)} = 0$$

Soln.

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$x^2 y'' + x y' + y = 0$$

Taking  $a_1 = x^2 y''$ ,  $a_2 = x y'$ ,  $a_3 = y$

Considering  $a_1$

$U = y''$	$V = x^2$	$U = y'$	$V = x$	$U = y$	$V = 1$
$U^{(n)} = y^{(n+2)}$	$V' = 2x$	$U^{(n)} = y^{(n+1)}$	$V' = 1$	$U^{(n)} = y^{(n)}$	$V' = 0$
$U^{(n-1)} = y^{(n+1)}$	$V'' = 2$	$U^{(n-1)} = y^{(n)}$	$V'' = 0$		
$U^{(n-2)} = y^{(n)}$	$V''' = 0$				

$$y^{(n)} = U^{(n)}V + n U^{(n-1)}V' + \frac{n(n-1)}{2!} U^{(n-2)}V'' + \frac{n(n-1)(n-2)}{3!} U^{(n-3)}V''' + \dots$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} U^{(n-4)}V^{(4)} + \dots$$

Therefore,

$$a_1^{(n)} = y^{(n+2)} \cdot x^2 + n [y^{(n+1)} \cdot 2x] + \frac{n(n-1)}{2!} \cdot y^{(n)} \cdot 2 + 0$$

$$a_2^{(n)} = y^{(n+1)} \cdot x + n (y^{(n)} \cdot 1) + 0$$

$$a_3^{(n)} = y^{(n)} \cdot 1 + 0$$

Substitute values of  $a_1^{(n)}$ ,  $a_2^{(n)}$ ,  $a_3^{(n)}$  into general eqn

$$x^2 y'' + x y' + y = 0$$