(i) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y''' = 2e^{x^2} + (2x+1)^2 e^{x^2+x}$

$y' (2x+1) + 2y$

$= (2x+1)e^{x^2+x} \cdot (2x+1) + 2(x^2+x)$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

from the equation above

part A

$A = y''$, $A' = y'''$, $A^n = y^{2+x}$

B

$B = y'(2x+1)$,

$u = y'$, $u^n = y^{n+1}$

$v = 2x+1$, $v' = 0$

$\therefore B^n = (y^{n+1})(2x+1) + n(y^n)(2) + 0$

$B^n = (2x+1)y^{n+1} + 2ny^n$

- C,

$C = 2y$

$C^n = 2y^n$

$A^n = B^n + C^n$

$\cancel{y^{n+2} = (2x+1)}$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

$y = x^3 e^{4x}$

let $u = e^4$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$ $u_n = 4^n e^{4x}$

let $v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 0$

By leibniz theorem

$y^n = 4^n e^{4x} . x^3 + n . 4^{n-1} e^{4x} . 3x^2 + n(n-1) .$
$4^{n-2} e^{4x} . 6x + \frac{n(n-1)(n-2)}{3!} . 4^n e^{4x} . \frac{6+0}{2!}$

$y^n = 4^n e^{4x} . x^3 + 3x^2 . 4^{n-1} e^{4x} + 3n(n-1).$
$- 4^{n-2} e^{4x} . x$ $n(n-1)(n-2) . 4^{n-3} e^{4x}$

$\therefore y^5 = 4^5 e^{4n} . x^3 + 3x + 3x^2 (5) : 4^4 e^{4x} + 3(5)(4)$
$4^3 e^{4x} . x + (5)(4)(3) . 4^2 e^{4x}$

$y^5 = 1024 e^{4x} . x^3 + 3440 e^{4!} x^2 + 3840 e^{4x} x$
$+ 960 e^{4x}$

$y^5 = 64 e^{4x} (16x^3) + 60x^2 + 60x + 15).$

iii $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

for $A = x^2 y''$

$u = y''$, $u^n = y^{n+2}$

$v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$A^n = (y^{n+2}) x^2 + n (y^{n+1}) . 2x + n \frac{(n-1)}{2!}.$
$(y^n) . 2 + 0$

$A^n = x^2 y^{(n+2)} + 2nxy^{(n+1)} + n(n-1) y^n$

for B

$B = xy'$

$u = y'$, $u^n = y^{n+1}$

$v = x$, $v' = 1$, $v'' = 0$

$B^n = (y^{n+1}) . x + n (y^n) . 1 + 0$

$= xy^{(n+1)} + ny^n$

for C

$C = y$

$C^n = y_n$

$\therefore A^n + B^n + C^n = 0$

$= x^2 y_{(n+2)} + 2x\,n\,y_{(n+1)} + (n^2 - n)\, y_n + 2x y_{(n+1)} + n y_n$

$+ y_n = 0$

$= \cancel{x^2 y} \; x^2 y_{(n+2)} + x y_{(n+1)} (2n+1) + y_n (n^2 - n + n \times 1) = 0$

$= x^2 y_{(n+2)} + (2n+1)\, x y_{(n+1)} + (n^2 + 1)\, y_n = 0$