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(1) $y = e^{x^2+x}$

Show that

$$y'' = y'(2x+1) + 2y$$

and hence prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$$

Soln

$$y = e^{x^2+x}$$

Differentiate $y = y'$ & y''

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x} \cdot 2$$

$$y'' = (2x+1)y' + y \cdot 2$$

$\therefore$ Therefore,

$$y'' = y'(2x+1) + 2y$$

Finding the nth derivative of y''

i.e $y^{(n+2)} = y^{(n+1)}(2x+1) + n y^{(n)} \cdot 2 + 2y^{(n)}$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$$

Therefore

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^{(n)}(n+1)$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$$

(2)

$y = x^3 e^{4x}$, determine $y^{(5)}$

soln

$y = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!}$

$u^{(n-3)} v''' + \frac{n(n-1)(n-2)\cdot(n-3)}{3!} u^{(n-4)} v^{(4)} + \cdots$

taking

$u = e^{4x}$ $\quad V = x^3$

$u^n = 4^n e^{4x}$ $\quad V' = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad V'' = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad V''' = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad V^{(4)} = 0$

$u^{(n-4)} = 4^{(n-4)} e^{4x}$ $\quad V^{(5)} = 0$

$u^{(n-5)} = 4^{(n-5)} e^{4x}$

sub values in general soln

$y^n = 4^n e^{4x} \cdot x^3 + n[4^{(n-1)} e^{4x} \cdot 3x^2] + \frac{n(n-1)}{2!} \cdot 4^{(n-2)} e^{4x} \cdot 6x + \cdots$

$\cdots + \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} \cdot 6 + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{(n-4)} e^{4x} \cdot 0$

$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + \frac{n(n-1) 6x}{2\times 1} \cdot 4^{(n-2)} e^{4x} + \frac{n(n-1)(n-2) 6}{3\times 2\times 1}$

$4^{(n-3)} e^{4x} + 0$

$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)} e^{4x} + 3n^2(n-1) 4^{(n-2)} e^{4x} + n(n-1)(n-2) 4^{(n-3)}$

$e^{4x} + 0$

$y^n = y^5$, i.e $n = 5$, sub value $n = 5$ into eqn above

$y^5 = x^3 4^{(5)} e^{4x} + 3(5) x^2 4^{(5-1)} e^{4x} + 3(5)(5-1) x\, 4^{(5-2)} e^{4x} + 5 (5-1)(5-2) 4^{(5-3)} e^{4x} + 0$

$y^5 = 1024 x^3 e^{4x} + 15(256) x^2 e^{4x} + 15(4)(64) x\, e^{4x} + 5(4)(3)(16) e^{4x} + 0$

$y^5 = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} e^{4x}\ 3840 x e^{4x} + 960 e^{4x} + 0$

(2ii)

$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

show that

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

Soln

$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2 y'' + x y' + y = 0$

Taking $a_1 = x^2 y''$, $a_2 = x y'$, $a_3 = y$

Considering a_1

$u = y''$ $v = x^2$ $u = y'$ $v = x$ $u = y$ $v = 1$

$u^{(n)} = y^{(n+2)}$ $v' = 2x$ $u^{(n)} = y^{(n+1)}$ $v' = 1$ $u^n = y^n$ $v' = 0$

$u^{(n-1)} = y^{(n+1)}$ $v'' = 2$ $u^{(n-1)} = y^n$ $v'' = 0$

$u^{(n-2)} = y^{(n)}$ $v''' = 0$

$y^{(n)} = u^{(n)} v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^{(4)} + \dots$

Therefore;

$a_1^{(n)} = y^{(n+2)} \cdot x^2 + n\left[y^{(n+1)} \cdot 2x\right] + \frac{n(n-1)}{2!} \cdot y^n \cdot 2 + 0$

$a_2^{(n)} = y^{(n+1)} \cdot x + n\left(y^n \cdot 1\right] + 0$

$a_3^{(n)} = y^n \cdot 1 + 0$

substitute values of $a_1^{(n)}$, $a_2^{(n)}$, $a_3^{(n)}$ into general eqn

$x^2 y'' + xy' + y = 0$