

$$A^{n+1} - B^n + C^n$$

$$y^{n+2} = (2n+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2n+1)y^{n+1} + 2y^n(n+1)$$

$$y^{(n+2)} = (2n+1)y^{(n+1)} + 2(n+1)y^n$$

2 Solution

$$i) y = x^3 e^{4x}$$

let $u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u''' = 4^3 e^{4x}$

let $v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v^{(4)} = 0$

Using Leibniz theorem,

$$y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} \cdot 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} \cdot 4^{n-3} e^{4x} \cdot 6 + 0$$

$$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 n \cdot 4^{n-1} e^{4x} + 3n(n-1) \cdot 4^{n-2} e^{4x} x + n(n-1)(n-2) \cdot 4^{n-3} e^{4x}$$

$$\therefore y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2(5) \cdot 4^4 e^{4x} + 3(5)(4) \cdot 4^3 e^{4x} \cdot x + (5)(4)(3) \cdot 4^2 e^{4x}$$

$$y^5 = 1024 e^{4x} \cdot x^3 + 5840 e^{4x} \cdot x^2 + 3840 e^{4x} \cdot x + 960 e^{4x}$$

$$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$$

ii) $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^n = 0$

Solution

For part A,

$$A = x^2 y''$$

$$u = y'' , \quad u^n = y^{n+2}$$

$$v = x^2 , \quad v' = 2x , \quad v'' = 2 , \quad v''' = 0$$

$$A^n = (y^{n+2})x^2 + n(y^{n+1}) \cdot 2x + \frac{n(n-1)}{2} \cdot (y^2) \cdot 2 + 0$$

$$A^n = x^2 y^{(n+2)} + 2xy^{(n+1)} + n(n-1)y^n$$

for Part B,

$$B = xy'$$

$$u = y' , \quad u^n = y^{n+1}$$

$$v = x , \quad v' = 1 , \quad v'' = 0$$

$$B^n = (y^{n+1}) \cdot x + n(y^n) \cdot 1 + 0$$

$$= xy^{n+1} + ny^n$$

for Part C,

$$C = y$$

$$C^n = y^n$$

$$\therefore A^n + B^n + C^n = 0$$

$$= x^2 y^{(n+2)} + 2xy^{(n+1)} + (n^2 - n)y^n + xy^{n+1} + ny^n + y^n = 0$$

$$= x^2 y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2 - n + n + 1) = 0$$

$$= x^2 y^{(n+2)} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$$

1. Solution

$$y = e^{x^2+x}$$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$$

$$y'(2x+1) + 2y$$

$$= (2x+1)e^{x^2+x} \cdot (2x+1) + 2(e^{x^2+x})$$

$$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$$

but $y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$$\therefore y'' = y'(2x+1) + 2y$$

from the equation:

Part A

$$A = y'' \quad , \quad A' = y''' \quad , \quad A'' = y^{(4)}$$

Part B

$$B = y'(2x+1)$$

$$u = y' \quad , \quad u^n = y^{n+1}$$

$$v = 2x+1$$

$$v' = 2$$

$$v'' = 0$$

$$\therefore B = (y^{n+1})(2x+1) + n(y^n)(2) + 0$$

$$B' = (2x+1)y^{n+1} + 2ny^n$$

Part C

$$C = 2y$$

$$C' = 2y'$$