2.)

i. $y = x^3 e^{4x}$, determine $y^{(5)}$

soln

$$y^n = U^n V + nU^{(n-1)}V' + \frac{n(n-1)}{2!}U^{(n-2)}V'' + \frac{n(n-1)(n-2)}{3!}U^{(n-3)}V''' + \frac{n(n-1)(n-2)(n-3)}{3!}U^{(n-4)}V^{(4)} + \dots$$

taking

$U = e^{4x}$	$V = x^3$
$U^n = 4^n e^{4x}$	$V' = 3x^2$
$U^{(n-1)} = 4^{(n-1)}e^{4x}$	$V'' = 6x$
$U^{(n-2)} = 4^{(n-2)}e^{4x}$	$V''' = 6$
$U^{(n-3)} = 4^{(n-3)}e^{4x}$	$V^{(4)} = 0$
$U^{(n-4)} = 4^{(n-4)}e^{4x}$	$V^{(5)} = 0$
$U^{(n-5)} = 4^{(n-5)}e^{4x}$	

Sub values in general soln

$$y^n = 4^n e^{4x} \cdot x^3 + n[4^{(n-1)}e^{4x} \cdot 3x^2] + \frac{n(n-1)}{2!} \cdot 4^{(n-2)}e^{4x} \cdot 6x + \dots$$

$$\dots + \frac{n(n-1)(n-2)}{3!}4^{(n-3)}e^{4x} \cdot 6 + \frac{n(n-1)(n-2)(n-3)}{4!}4^{(n-4)}e^{4x} \cdot 0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)}e^{4x} + \frac{n(n-1)}{2\times1}6x \cdot 4^{(n-2)}e^{4x} + \frac{n(n-1)(n-2)}{3\times2\times1} \cdot 6 \cdot 4^{(n-3)}e^{4x} + 0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)}e^{4x} + \frac{n(n-1)}{\cancel{2}1}\overset{3}{\cancel{6}}x \cdot 4^{(n-2)}e^{4x} + \frac{\cancel{6}n(n-1)(n-2)}{\cancel{6}} \cdot 4^{(n-3)}e^{4x} + 0$$

$$y^n = x^3 4^n e^{4x} + 3nx^2 4^{(n-1)}e^{4x} + \overset{3nx}{\cancel{3x}}(n-1)4^{(n-2)}e^{4x} + n(n-1)(n-2)4^{(n-3)}e^{4x} + 0$$

$y^n = y^5$, i.e $n = 5$, Sub value $n=5$ into eqn above

$$y^5 = x^3 4^{(5)}e^{4x} + 3(5)x^2 4^{(5-1)}e^{4x} + \overset{3(5)(5-1)x}{\cancel{3(5)(5-1)}}4^{(5-2)}e^{4x} + 5(5-1)(5-\cancel{2})4^{(5-3)}e^{4x} + 0$$

$y^5 = 1024$

$y^5 = 10$

$x^2 \frac{d^2}{dx}$

Show the

$x^2 y^{(n+}$

$x^2 \frac{d^2y}{dx^2}$

$x^2 y'' +$

Considering:

$U = y''$

$U^{(n)} = y^{(n+2}$

$U^{(n-1)} = y^{(}$

$U^{(n-2)} = y$

$y^{(n)} = U^{(n)}V$

$\dots + n($

Therefore;

$a_1^{(n)} =$

$a_2^{(n)} = y$

$a_3^{(n)} =$

Subs

$(n-2)$.

21

$$y^5 = 1024x^3e^{4x} + 15(256)x^2e^{4x} + \cancel{15(4)(64)e^{4x}} \overset{15(4)(64)\,x\,e^{4x}}{} + 5(4)(3)(16)e^{4x} + 0$$

$$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x} + 0$$

(2ii)

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Show that

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$

Soln.

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$$x^2y'' + xy' + y = 0$$

Taking $a_1 = x^2y''$, $a_2 = xy'$, $a_3 = y$

Considering: a_1 a_2 a_3

$U = y''$ $V = x^2$ $U = y'$ $V = x$ $U = y$ $V = 1$

$U^{(n)} = y^{(n+2)}$ $V' = 2x$ $U^{(n)} = y^{(n+1)}$ $V' = 1$ $U^n = y^n$ $V' = 0$

$U^{(n-1)} = y^{(n+1)}$ $V'' = 2$ $U^{(n-1)} = y^n$ $V'' = 0$

$U^{(n-2)} = y^{(n)}$ $V''' = 0$

6.

$$y^{(n)} = U^{(n)}V + nU^{(n-1)}V' + \frac{n(n-1)}{2!}U^{(n-2)}V'' + \frac{n(n-1)(n-2)}{3!}U^{(n-3)}V''' + \dots$$

$$\dots + \frac{n(n-1)(n-2)(n-3)}{4!}U^{(n-4)}V^{(4)} + \dots$$

Therefore;

$$a_1^{(n)} = y^{(n+2)}\cdot x^2 + n[y^{(n+1)}\cdot 2x) + \frac{n(n-1)}{2!}\cdot y^n\cdot 2 + 0$$

$$a_2^{(n)} = y^{(n+1)}\cdot x + n(y^n\cdot 1] + 0$$

$$a_3^{(n)} = y^n\cdot 1 + 0$$

Substitute values of $a_1^{(n)}, a_2^{(n)}, a_3^{(n)}$ into general eqn

$$x^2y'' + xy' + y = 0$$

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i) $y = e^{x^2+x}$

Show that $y'' = y'(2x+1) + 2y$, Hence Prove that
$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2(n+1)\,y^{n}$

Sln

$y = e^{x^2+x}$

Differentiate $y = y'$ & y''

$y' = (2x+1)\,e^{x^2+x}$

$y'' = (2x+1)(2x+1)\,e^{x^2+x} + e^{x^2+x}\,2$

$y'' = (2x+1)\,y' + y \cdot 2$

Therefore

$y'' = y'(2x+1) + 2y$

finding the nth derivative of y''

i.e $y^{(n+2)} = y^{(n+1)}(2x+1) + ny^{(n)} \cdot 2 + 2y^{(n)}$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}(n+1).$

Therefore

$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2y^{(n)}(n+1)$

$y^{(n+2)} = (2x+1)\,y^{(x+)} + 2(n+1)\,y^{(n)}.$