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17/ENG06/024
Ed. MECHANICAL ENGINEERING
ASSIGNMENT 2

(1.) if $y = e^{x^2+x}$
Show that
$y'' = y'(2x+1)+2y$
$y^{(n+2)} = 2(x+1)y^{(n+1)} + 2(n+1)y^n$

Solution

(1.) $y = e^{x^2} + x$
$y' = (2x+1)e^{x^2+x}$
$y'' = (2x+1)(2x+1)e^{x^2+x} + 2 \cdot e^{x^2+x}$
Considering equation (i) and (ii)
$y'' = (2x+1)y' + 2y$

(2.) From the equation above it has been proven that
$y'' = y'(2x+1) + 2y$
let, $y'' = w_1$
$y'(2x+1) = w_2$
$2y = w_3$
from $w_1 = y''$
$u = y''$. $v = 1$
$u^n = y^{n+2}$ $v = 0$

From $w_1 = y''$
$u = y''$ $v = 1$
$u^n = y^{n+2}$ $v = 0$
From $w_2 = y'(2x+1)$
$u = y'$ $v = 2x+1$
$u^n = y^{n+1}$ $v' = 2$
$u^{n-1} = y'$ $v'' = 0$

From $w_3 = 2y$

$u^n = y^n \quad V = 2$

$u^{(n-1)} = y^n \quad V' = 0$

$w_1 + w_2 + w_3$

Using The ~~formula~~ Leibnitz formular

$y^n = u^n V + n u^{n-1} V' + \frac{n(n-1)}{2!} u^{(n-2)} V^{(2)}$

$w_1 = y^{n+2} \cdot 1$

$w_2 = y^{(n+1)}(2x+1) + n y^n \cdot 2$

$= y^{(n+1)}(2x+1) + 2n y^n$

$w_3 = y^n \cdot 2$

$w_1 = w_2 + w_3$

$y^{n+2} = y^{(n+1)}(2x+1) + 2n y^n + 2y^n$

$y^{n+2} = y^{(n+1)}(2x+1) + y^n(2n+2)$

$y^{n+2} = y^{n+1}(2x+1) + 2(n+1) y^n$

Q.E.D.

Question 2

(1) $y = x^3 e^{4x}$ Determine $y^{(5)}$

$u = e^{4x} \quad V = x^3$

$u^n = 4^n e^{4x} \quad V' = 3x^2$

$u^{n-1} = 4^{(n-1)} e^{4x} \quad V'' = 6x$

$u^{n-2} = 4^{(n-2)} e^{4x} \quad V''' = 6$

$u^{n-3} = 4^{(n-3)} e^{4x} \quad V^{iv} = 0$

Using Leibnitz formular

$y \Rightarrow u^n V + n V^{n-1} V' + \frac{n(n-1)}{2!} u^{(n-2)} V^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} V^3 + \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} V^4$

$y^n \Rightarrow 4^n e^{4x} \cdot x^3 + n(4^{(n-1)} e^{4x}) \cdot 3x^2 + \frac{n(n-1)}{2!}(4^{(n-2)} e^{4x}) \cdot 6x + \frac{n(n-1)(n-2)}{3!}$

$(4^{(n-3)} e^{4x}) \cdot 6 + 0$

$y^n = 4^n e^{4x} \cdot x^3 + n(4^{(n-1)} e^{4x}) \cdot 3x^2 + n(n-1)(4^{(n-2)} e^{4x}) \cdot 3x + n(n-1)(n-2)$
$(4^{(n-3)} e^{4x})$

$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 \cdot n(4^{(n-1)} e^{4x}) + 3x\, n(n-1)(4^{(n-2)} 4x) + n(n-1)(n-2)$
$(4^{(n-3)} e^{4x})$

$y^{(5)} = x^3 4^5 e^{4x} + 3x^2 \cdot 5(4^{(5-1)}) 4x + 3x(5(5-1)) 4^{(5-2)} e^{4x} + 5(5-1)$
$(5-2) \cdot (4^{(5-3)} e^{4x}$

$y^5 = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

(2) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2 y'' + x y' + y = 0$

$w_1 = x^2 y''$

$w_2 = x y'$

$w_3 = y$

From ; $w_1 = x^2 y''$

$u = y''$; $v = x^2$

$u^n = y^{n+2}$ $\quad v' = 2x$

$u^{n-1} = y^{n+1}$ $\quad v'' = 2$

$u^{n-2} = y^n$ $\quad v''' = 0$

from; $w_2 = xy'$. $\quad u = y'$, $v = x$

$u^n = y^{n+1}$ $\quad v' = 1$

from; $w_3 = y$; $u = y$ $\quad v = 1$

$u^n = y^n$. $\quad v' = 0$

Using Leibnitz theorem

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$$= u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} (u^{(n-2)}) v'' + \frac{n(n-1)(n-2)}{3!} (u^{n-3}) v''' \ldots$$

$$\omega_1 = y^{(n)} = y^{(n+2)} x^2 + n y^{(n+1)} 2x + \frac{n(n-1)}{2!} y^n \cdot 2$$

$$\omega_2 = y^{(n+1)} x + n y^n$$

$$\omega_3 = y^n \cdot 1$$

$$\omega_1 + \omega_2 + \omega_3$$

$$= y^{(n+2)} x^2 + n y^{(n+1)} 2x + n(n-1) y^n + y^{(n+1)} x + n y^n + y^{(n)}$$

$$= x^2 y^{(n+2)} + x y^{n+1} (2n+1) + (\cancel{n^2 + n - n} + 1) y^{(n)}$$

$$= x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + (n^2 + 1) y^{(n)}$$